Mathematics for Engineers I
Trigonometry

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Week Six

Birmingham City University, Engineering and Built Environment

BIRMINGHAM CITY
University

What will you learn

These lessons will be mostly, obviously, maths :

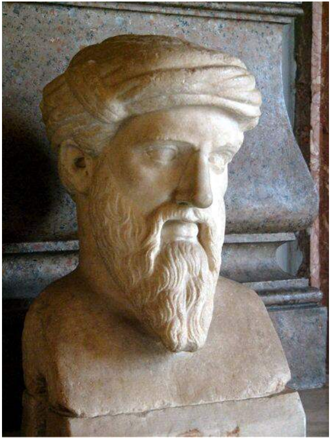
- what is *trigonometry*
- how it works - practice !
- why it is useful - physics and engineering

I will try to explain in detail (maybe too much). *Please* let me know if you do not understand something.

Pythagoras

" There is geometry in the humming of the strings. "

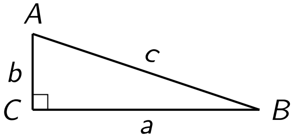
Pythagoras



Pythagoras theorem

The Greek mathematician Pythagoras discovered that :

- in a right angled triangle
- the square of the hypotenuse is equal to the sum of the squares of the other two sides.



Note that the hypotenuse is *the longest side* (c here).

$$a^2 + b^2 = c^2$$

Pythagoras theorem

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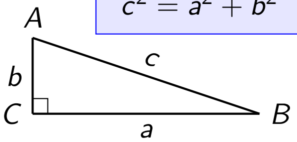
- in a right angled triangle
- the ne
  squa

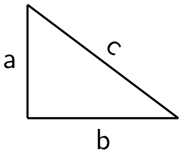
Pythagoras theorem :
if c it the hypotenuse in a right triangle then
$c^2 = a^2 + b^2$



Note that the hypotenuse is *the longest side* (c here).

$$a^2 + b^2 = c^2$$

proof of the theorem



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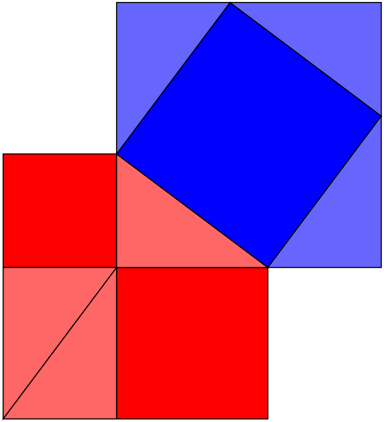

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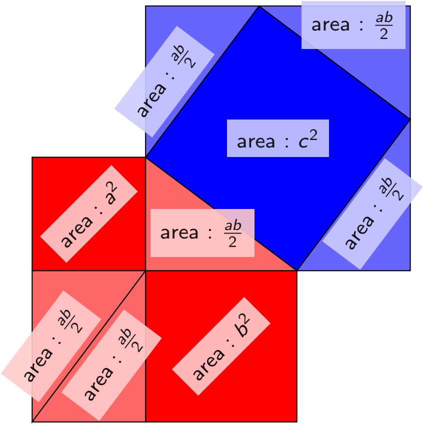


the blue area = the red area

Let's name a the side of the small square, b of the medium one and c of the large one. The red area is

$$a^2 + b^2 + 3 \times \frac{ab}{2}$$

The blue area is

$$c^2 + 3 \times \frac{ab}{2}$$

It means that

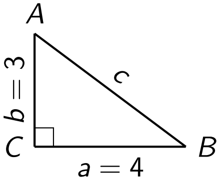
$$c^2 = a^2 + b^2$$

Consequence

If you know two sides of a right triangle, you can calculate the last one !

What is c ?
ABC is rectangle. The hypothenuse is c, and $a = 4$, $b = 3$. Pythagoras theorem : $a^2 + b^2 = c^2$

$$\begin{aligned} c^2 &= 4^2 + 3^2 \\ &= 16 + 9 \\ &= 25 \end{aligned}$$



hence

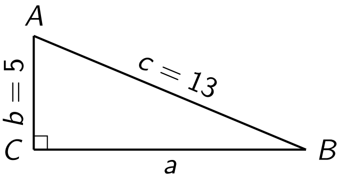
$$c = \sqrt{25} = 5$$

Consequence

If you know two sides of a right triangle, you can calculate the last one !

What is a ?
ABC is rectangle and $b = 5, c = 13$. The hypotenuse is c : $a^2 + b^2 = c^2$
We rearrange the terms :

$$a^2 = c^2 - b^2$$



and

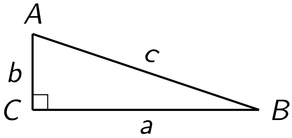
$$\begin{aligned} a^2 &= 13^2 - 5^2 \\ &= 169 - 25 \\ &= 144 \end{aligned}$$

and hence,

$$a = \sqrt{144} = 12$$

Trigonometric functions

Trigonometric functions are functions that **depend** on an angle.



- $\sin \widehat{A} = \frac{a}{c} = \frac{\text{opposite}}{\text{hypothenuse}}$
- $\cos \widehat{A} = \frac{b}{c} = \frac{\text{adjacent}}{\text{hypothenuse}}$
- $\tan \widehat{A} = \frac{a}{b} = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin \widehat{A}}{\cos \widehat{A}}$

Note that Sin is short for Sine, Cos is short for Cosine and Tan is short for Tangent

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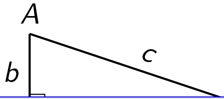

These ratios allow both to
⇝ define the trigonometric functions such as Sine
⇝ calculate the angle

- $\cos \widehat{A} = \dfrac{b}{c} = \dfrac{\text{adjacent}}{\text{hypothenuse}}$
- $\tan \widehat{A} = \dfrac{a}{b} = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{\sin \widehat{A}}{\cos \widehat{A}}$

Note that Sin is short for Sine, Cos is short for Cosine and Tan is short for Tangent

What to do in general ?

- Draw and label a picture
  including the sides and the angle θ. A variable should represent the unknown.
- Identify which sides are labeled, using the terms opposite, adjacent and hypotenuse.
  use the viewpoint of the angle of interest θ

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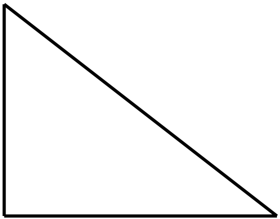
- Write the proper equation.
- Substitute the values you know for the sides and angles.
- Solve for the variable using algebra.

Let's focus on the triangle

- Draw the triangle :



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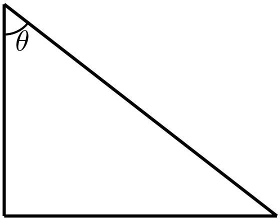
- If needed, locate the angle of interest



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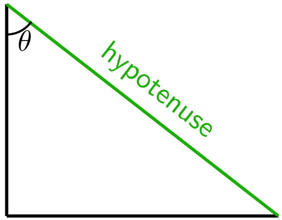
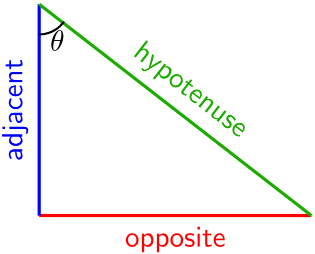
- Locate the hypotenuse



Let's focus on the triangle

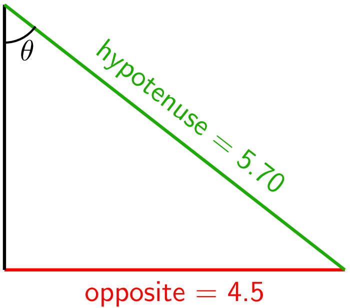
- Draw the triangle :
- If needed, locate the angle of interest
- Locate the hypotenuse
- Locate the adjacent and opposite sides



sine

It is the $\frac{\text{opposite side}}{\text{hypothenuse}}$.

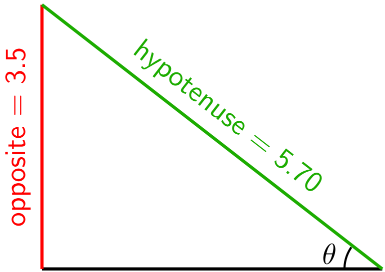
$$\sin\theta = \frac{4.5}{5.70} = 0.789$$



sine

It is the $\frac{\textit{opposite side}}{\textit{hypothenuse}}$.

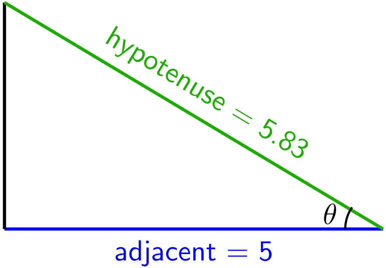
$$\sin\theta = \frac{3.5}{5.70} = 0.858$$



cosine

It is the $\frac{\textit{adjacent side}}{\textit{hypothenuse}}$.

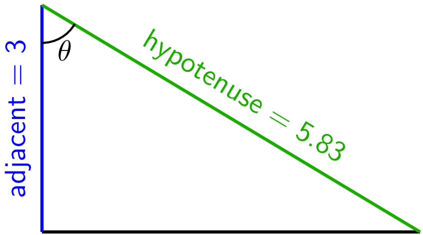
$$\cos\theta = \frac{5}{5.83} = 0.858$$



cosine

It is the $\frac{\textit{adjacent side}}{\textit{hypothenuse}}$.

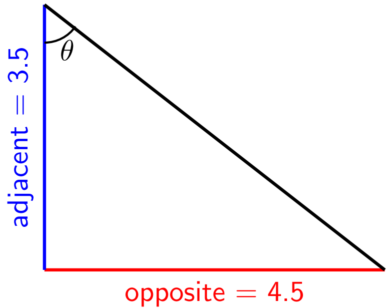
$$\cos\theta = \frac{3}{5.83} = 0.515$$



tangent

It is the $\frac{\textit{opposite side}}{\textit{adjacent side}}$.

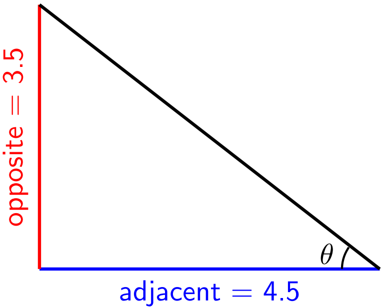
$$\tan\theta = \frac{4.5}{3.5} = 1.286$$



tangent

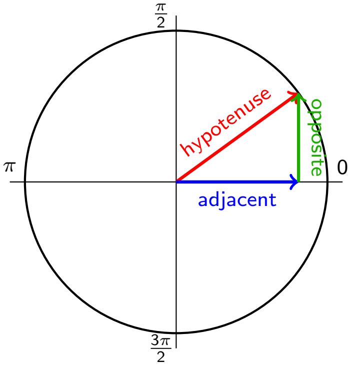
It is the $\frac{\textit{opposite side}}{\textit{adjacent side}}$.

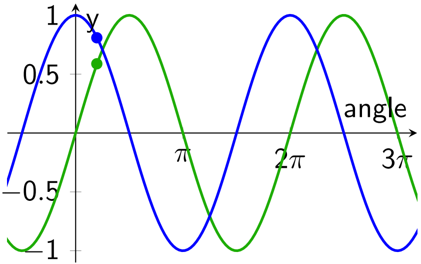
$$\tan\theta = \frac{3.5}{4.5} = 0.778$$



Periodicity ?

the function sine and cosine are periodic. Why ? Because they are related to a circle





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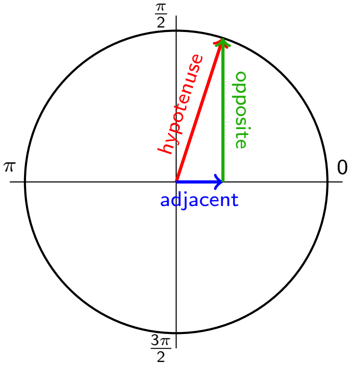

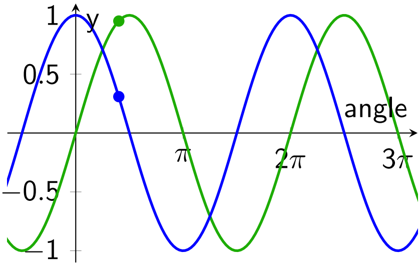

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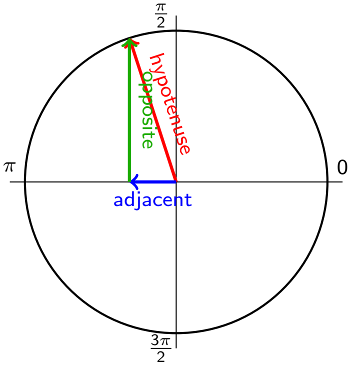

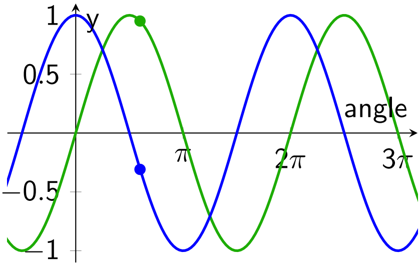

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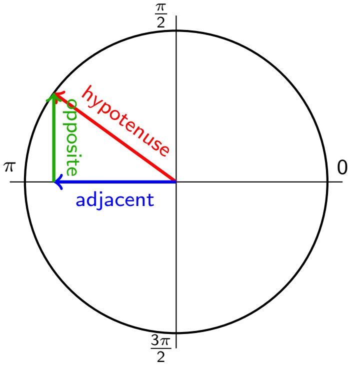

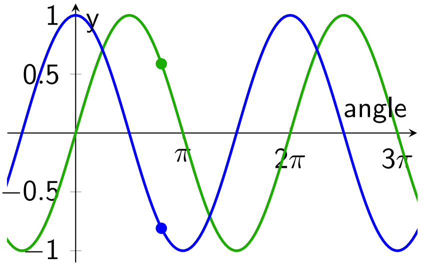

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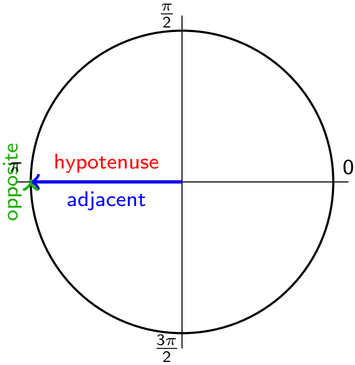

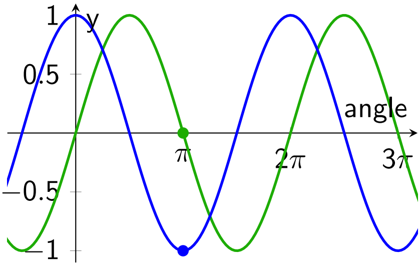

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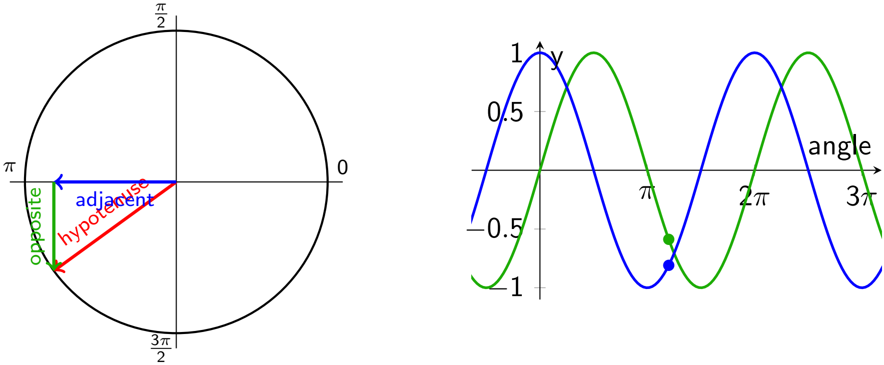

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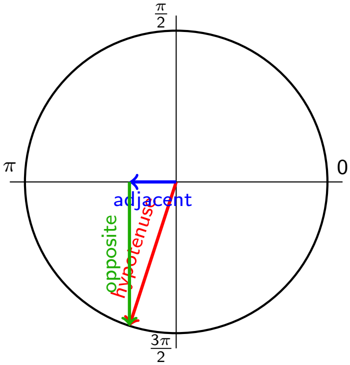

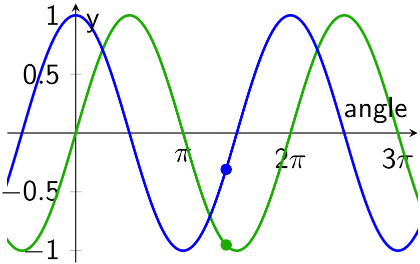

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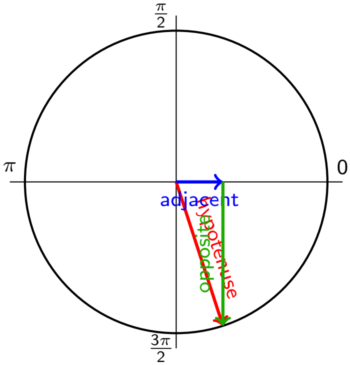

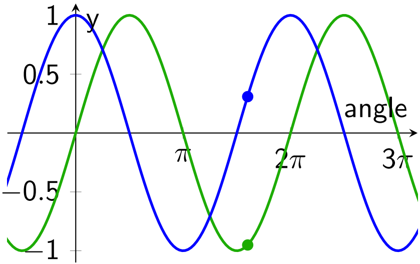

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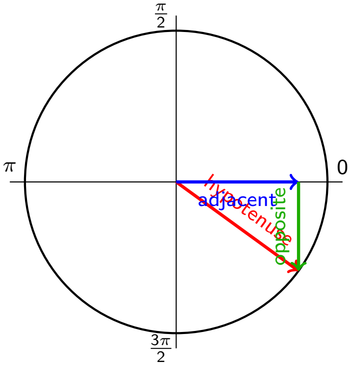

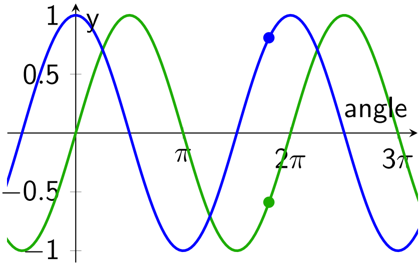

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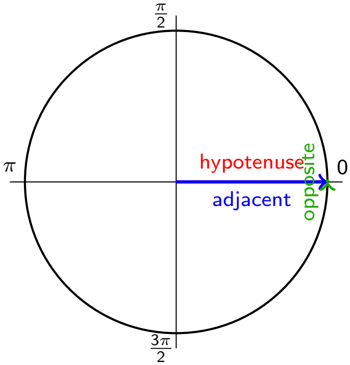

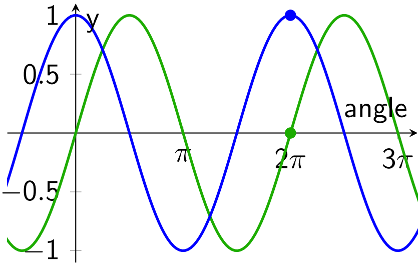

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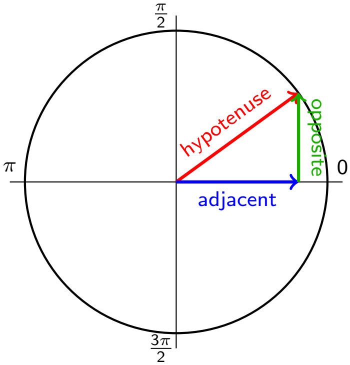
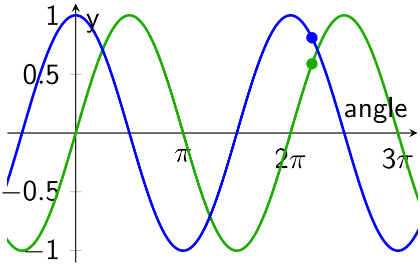


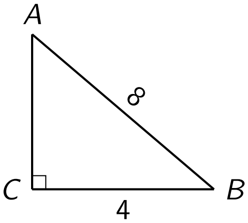


Example : angles

What is the angle $\widehat{A}$?

Which one of the 3 ratios to use ?

- Sine
- Cosine
- Tangent



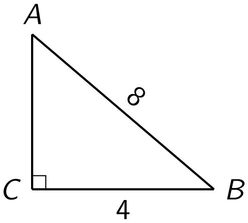
Example : angles

What is the angle $\widehat{A}$?

In relation to A :

- the 4 is the opposite side
- the 8 is the hypotenuse

$\rightsquigarrow$ Opposite and hypotenuse means sin.



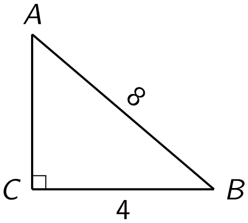
Example : angles

What is the angle $\widehat{A}$?

$$\sin \widehat{A} = \frac{\text{opposite}}{\text{hypothenuse}} = \frac{4}{8} = 0.5$$

So $\sin \widehat{A} = 0.5$.
How can we calculate $\widehat{A}$? $\rightsquigarrow$ *calculator*.

$$\widehat{A} = \text{Inv Sin}(0.5) = 30^o$$



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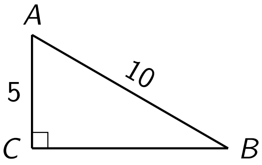


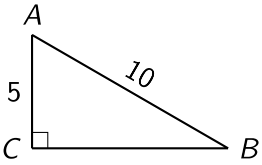
Example : angles

What is the angle $\widehat{A}$?

In relation to A :

- the 5 is the adjacent side
- the 10 is the hypotenuse

$\rightsquigarrow$ adjacent and hypotenuse means cos.



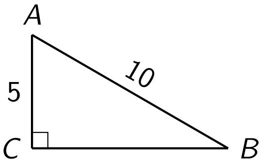
Example : angles

What is the angle $\widehat{A}$?

$$\cos \widehat{A} = \frac{\text{adjacent}}{\text{hypothenuse}} = \frac{5}{10} = 0.5$$

So $\cos \widehat{A} = 0.5$.

$$\widehat{A} = \cos^{-1}(0.5) = 60^o$$

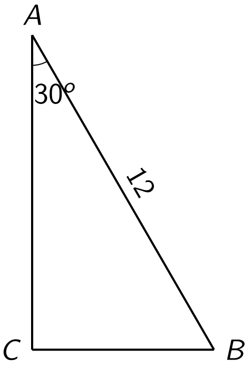


Example : side

What is the side AC ?

Which one of the 3 ratios to use ?

- Sine
- Cosine
- Tangent



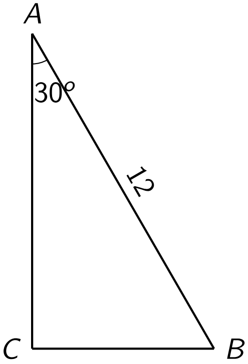
Example : side

What is the side AC ?

we have or want :

- the angle ($\widehat{A} = 30^0$)
- the hypotenuse ($AB = 12$)
- the adjacent side (AC)

Adjacent and hypotenuse $\rightsquigarrow$ cos.



Example : side

What is the side AC ?

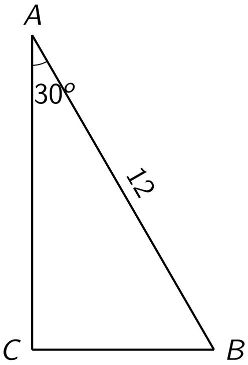
$$\cos\widehat{A} = \frac{adjacent}{hypothenuse} = \frac{AC}{AB}$$

and :

$$AC = \cos\widehat{A} \times AB$$

it follows

$$\begin{aligned} AC &= \cos\widehat{A} \times AB \\ &= \cos(30^o) \times 12 \\ &= 0.866 \times 12 \\ &= 10.39 \end{aligned}$$

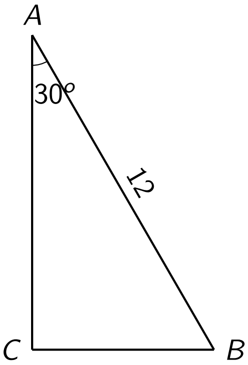


Example : side

What if we want BC ?

Which one of the 3 ratios to use ?

- Sine
- Cosine
- Tangent



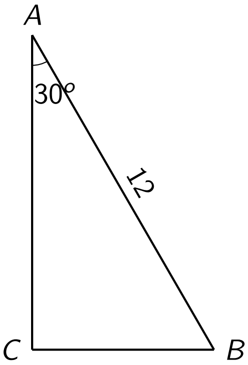
Example : side

What if we want BC ?

we have or want :

- the angle ($\widehat{A} = 30^0$)
- the hypotenuse ($AB = 12$)
- the opposite side (BC)

Opposite and hypotenuse $\rightsquigarrow$ sin.



Example : side

What if we want BC ?

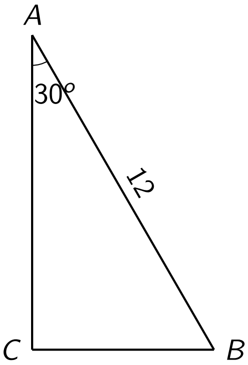
$$\sin \widehat{A} = \frac{opposite}{hypothenuse} = \frac{BC}{AB}$$

and :

$$BC = \sin \widehat{A} \times AB$$

it follows

$$\begin{aligned} BC &= \sin \widehat{A} \times AB \\ &= \sin(30^o) \times 12 \\ &= 0.5 \times 12 \\ &= 6 \end{aligned}$$

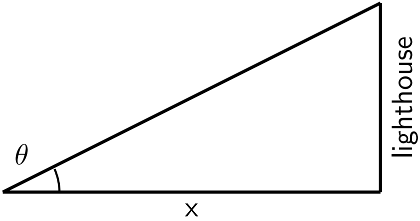


the lighthouse

A man is in a boat. He sees the light of the Flamborough Head Lighthouse, which, as everybody knows, culminates at 65m.
The saiman sees i at an angle of 20 degrees above the horizon.
How far away from the shore is the boat ?

wiki commons

the lighthouse

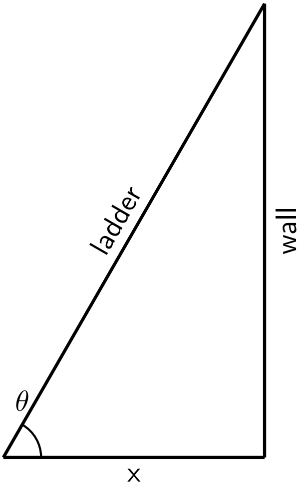


ladder

- A ladder placed against a wall such that it reaches the top of the wall.
- The height of the wall is 6m.
- The ladder is inclined at an angle of 60 degree.

How far the ladder is from the foot of the wall ?

ladder

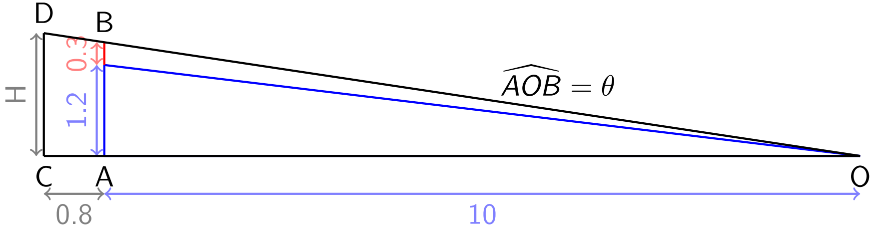


stadium

How to build a stadium ?
Every spectator has to be able to see the same thing :

- For the first row, the spectator is supposed to see well at 10m.
- Also, when seated, the spectator eye-level is 1.2m.
- The second row is 0.8m behind.
- For being sure that there is no issue with the view, there is an extra 0.3m of clearance from his eye-level to the second spectator's line-of-sight.

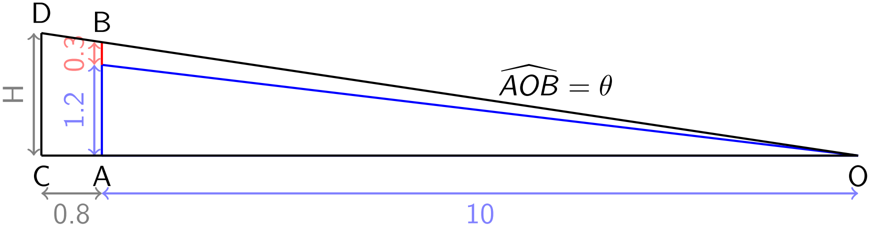
stadium



Let's draw everything. We have :

- The first spectator is seated in A, the second in C.
- B is the view going on top of the first spectator
- D is the eye level of the second spectator.

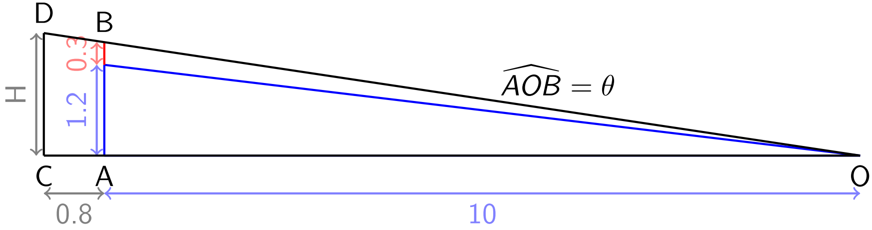
stadium



For calculating DC, we need $\theta = \widehat{AOB} = \widehat{COD}$:

$$\tan \widehat{AOB} = \frac{AB}{AO} = \tan \widehat{COD} = \frac{DC}{CO}$$

stadium



It means that

$$H = DC = CO \tan\theta = CO\frac{AB}{AO}$$

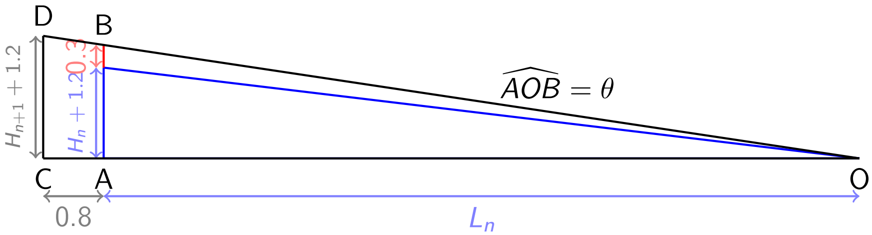
or

$$H = (10 + 0.8) \times \frac{1.2 + 0.3}{10}$$

The seat has to be $H - 1.2 = 0.42m$ higher in the second row.

and if we continue ?

We can iterate to have the third seat, etc.

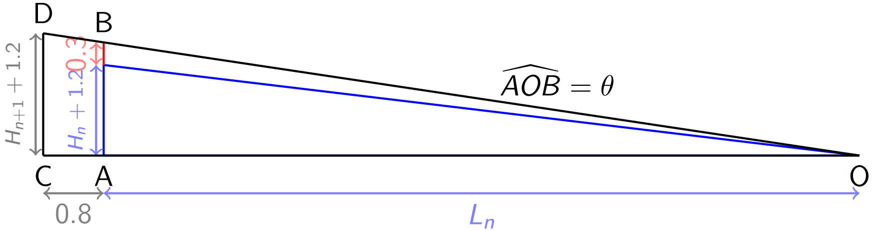


Let's draw everything. We have :

- The *n*th spectator is seated in A, at an height *h*.
- The $n+1$th is seated in C.
- B is the view going on top of the *n*th spectator
- D is the eye level of the $n+1$th spectator.

and if we continue ?

We can iterate to have the third seat, etc.



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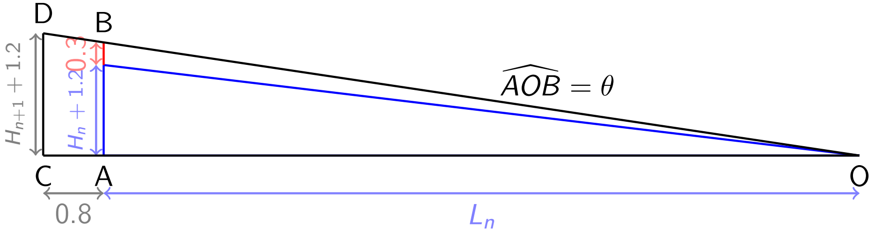
and if we continue ?

We can iterate to have the third seat, etc.



It means that

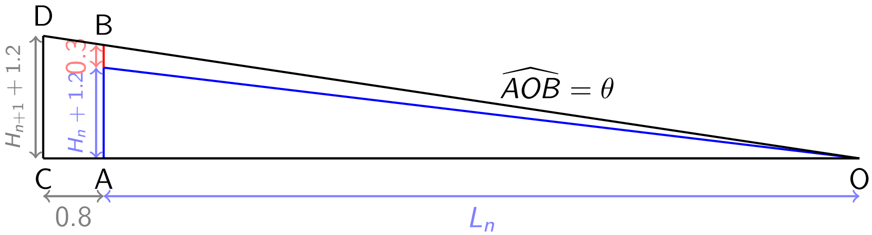
$$H_{n+1} = DC = CO \tan\theta = CO\frac{AB}{AO}$$

or

$$H_{n+1} = L_{n+1} \times \frac{1.2 + 0.3 + H_n}{L_n} - 1.2$$

and if we continue ?

We can iterate to have the third seat, etc.



For instance, the third seat has to be at the height $H_3 = L_3 \dfrac{H_2 + 1.5}{L_2} = 11.6 \times \dfrac{0.42 + 1.5}{10.8} - 1.2 = 0.86m$.

and if we continue ?

Let's compare with the St Andrews Stadium !