Mathematics for Engineers I
Trigonometry

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Week Six
Birmingham City University, Engineering and Built Environment

BIRMINGHAM CITY
University

What will you learn

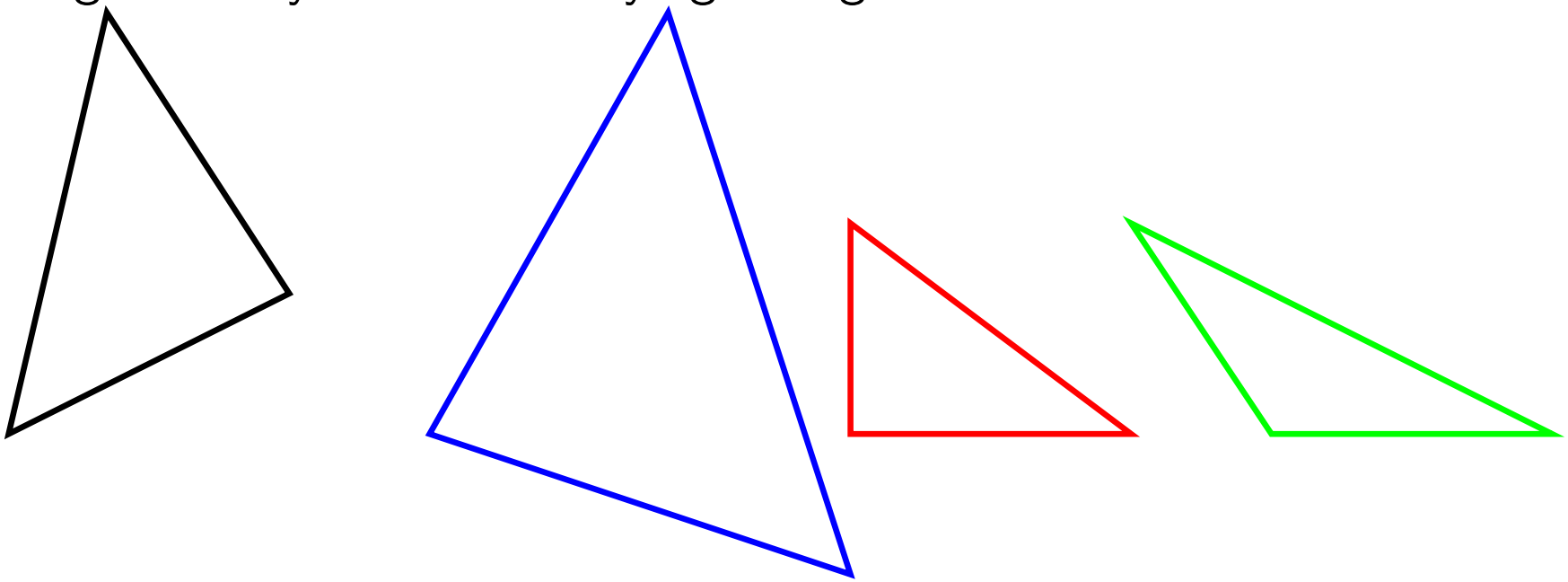
These lessons will be mostly, obviously, maths :

- what is *trigonometry*
- how it works - practice !
- why it is useful - physics and engineering

I will try to explain in detail (maybe too much). *Please* let me know if you do not understand something.

Triangles

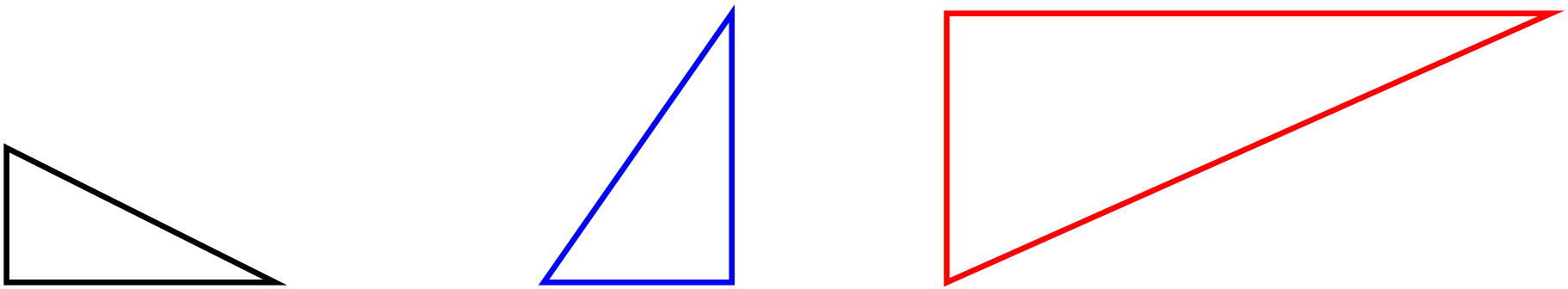
Trigonometry is about studying triangles.



A triangle is a shape with three edges.

A rectangle triangle

Today, we will focus on *rectangle triangles*.
This is a triangle where one of the angles is a right angle.



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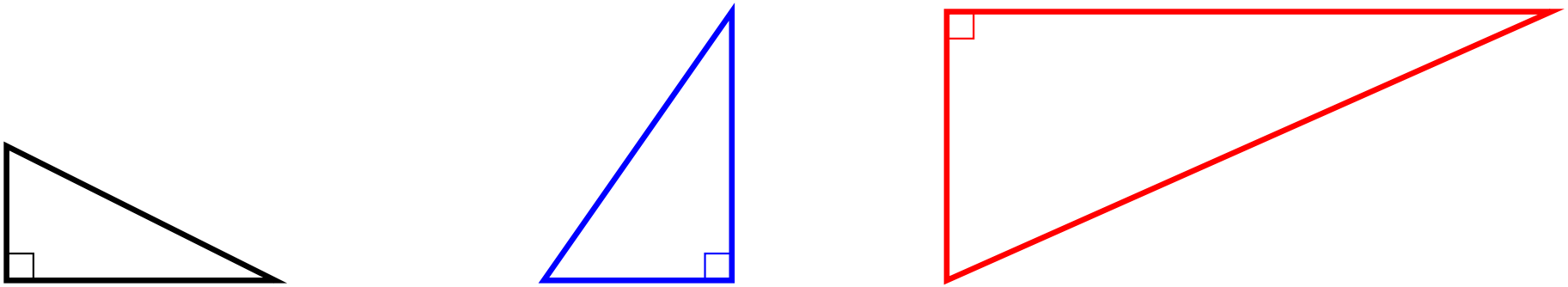


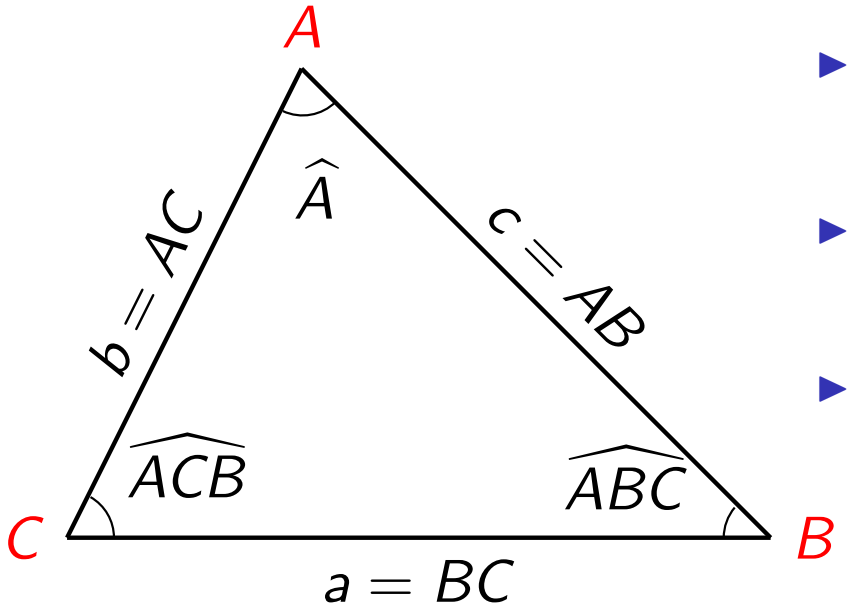
The right angle is usually designated with a □

the right angle

The measure of the right angle is equal to

- 90° in degrees
- $\frac{\pi}{2}$ in radiant

Reminders



We will use the notations :

- the vertices with capital letters
- the sides with lower case letters
- Side a is opposite the angle $\widehat{A}$ and so on.
- the distance between two points A and B : AB.
- the angle between the line (BC) and the line (AC) either :
  - $\widehat{BCA}$
  - $\widehat{ACB}$
  - $\widehat{C}$

    note that C, as the summit, is always in the middle

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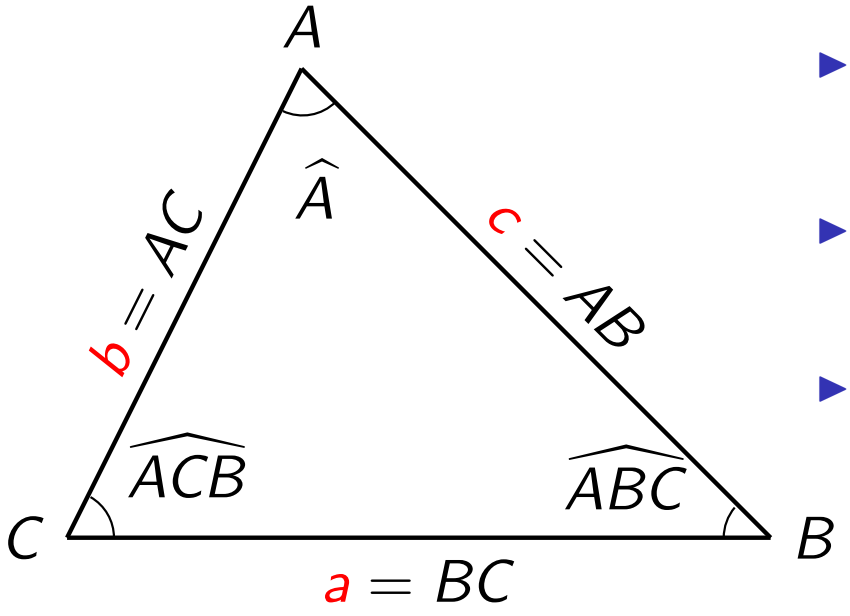

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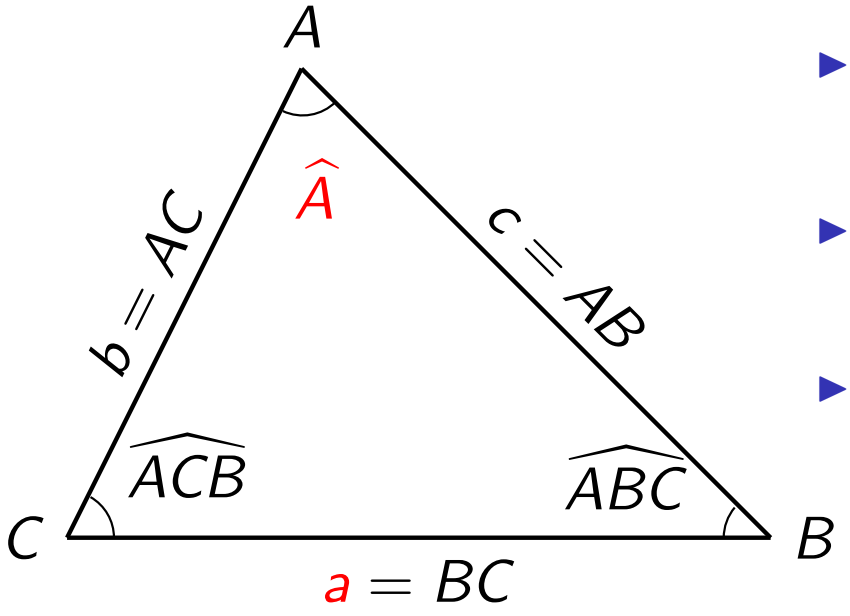

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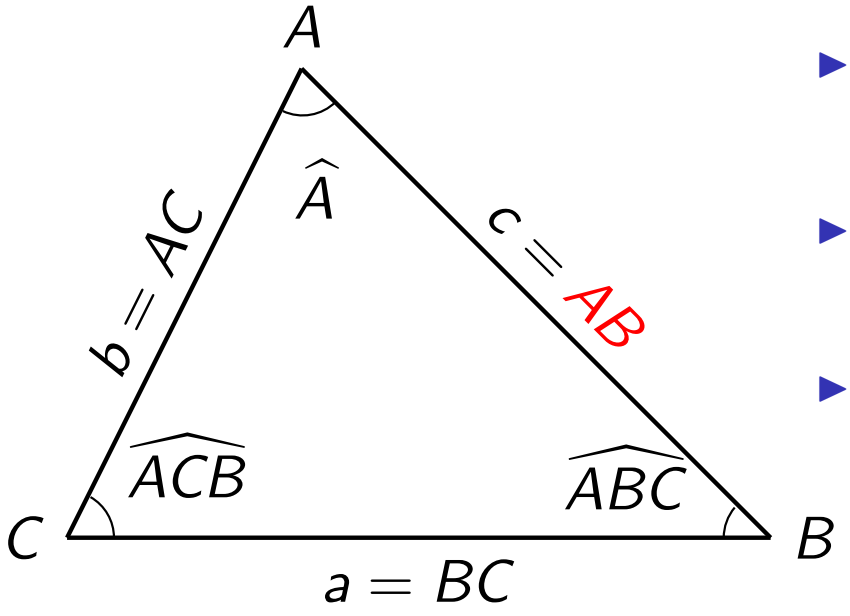

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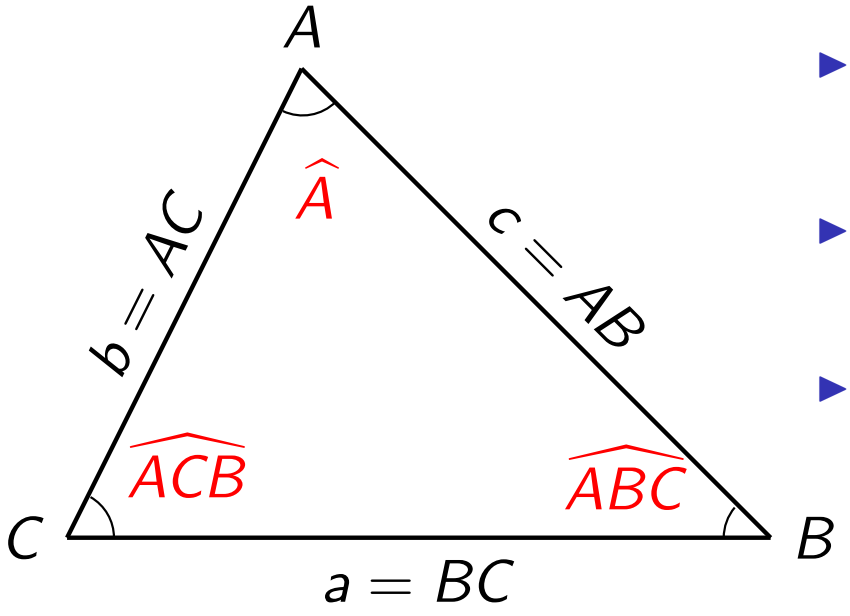

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Some chocolate ?

You can take the first two pieces of chocolate, or the third one.
Which one will you choose ?

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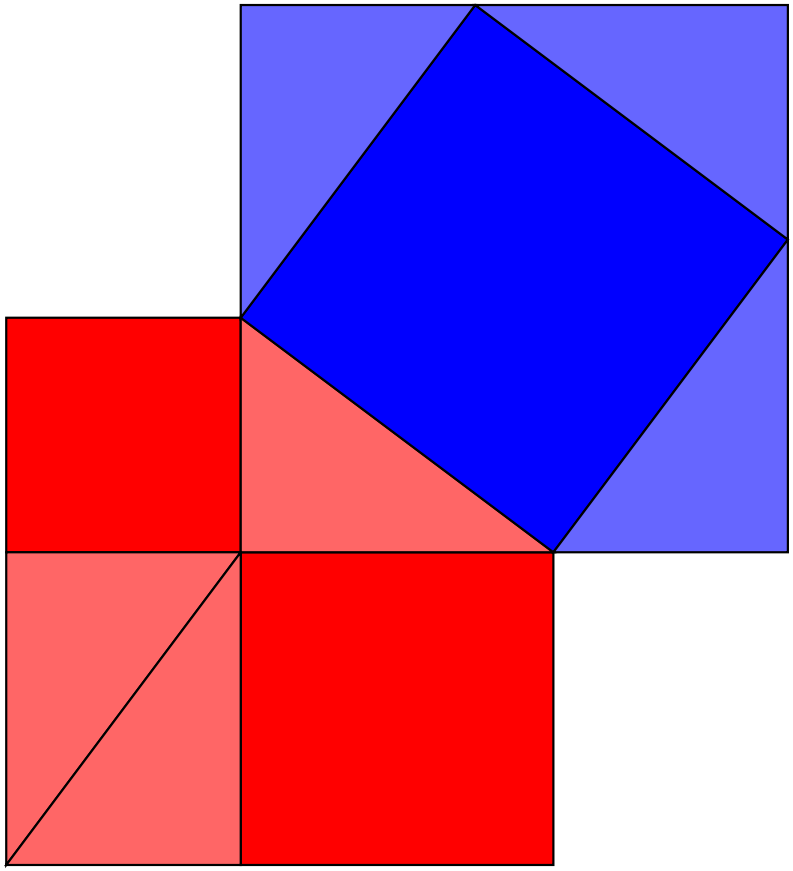

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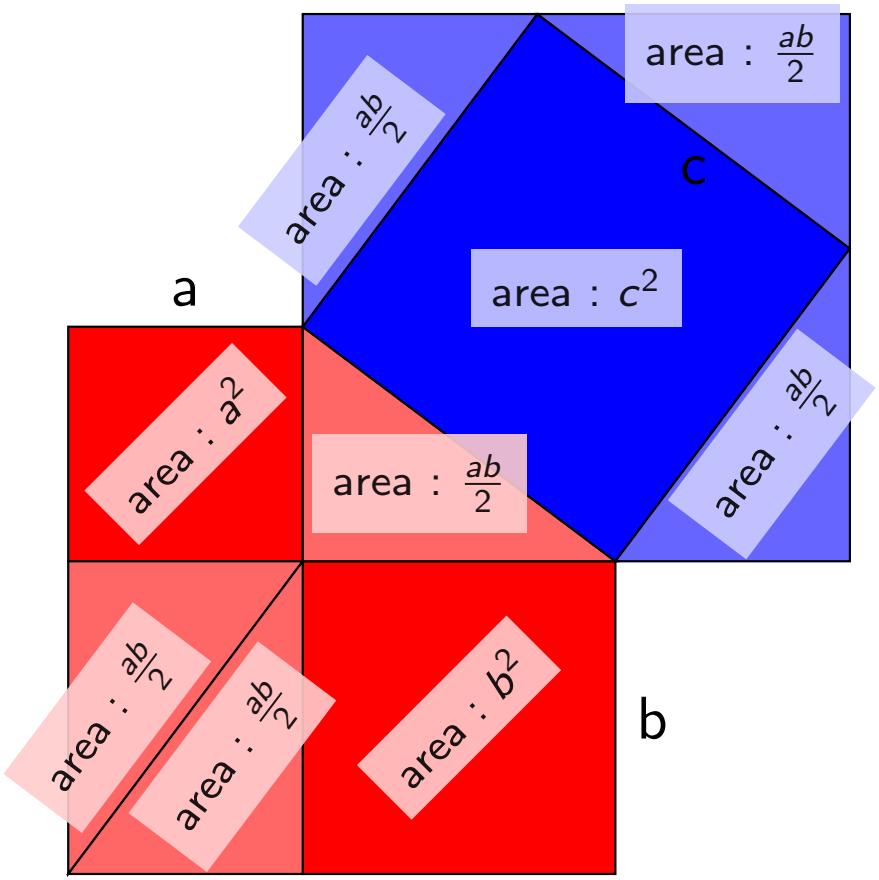

the blue area = the red area

Let's name a the side of the small square, b of the medium one and c of the large one. The red area is

$$a^2 + b^2 + 3 \times \frac{ab}{2}$$

The blue area is

$$c^2 + 3 \times \frac{ab}{2}$$

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Let's name a the side of the small square, b of the medium one and c of the large one. The red area is

$$a^2 + b^2 + 3 \times \frac{ab}{2}$$

The blue area is

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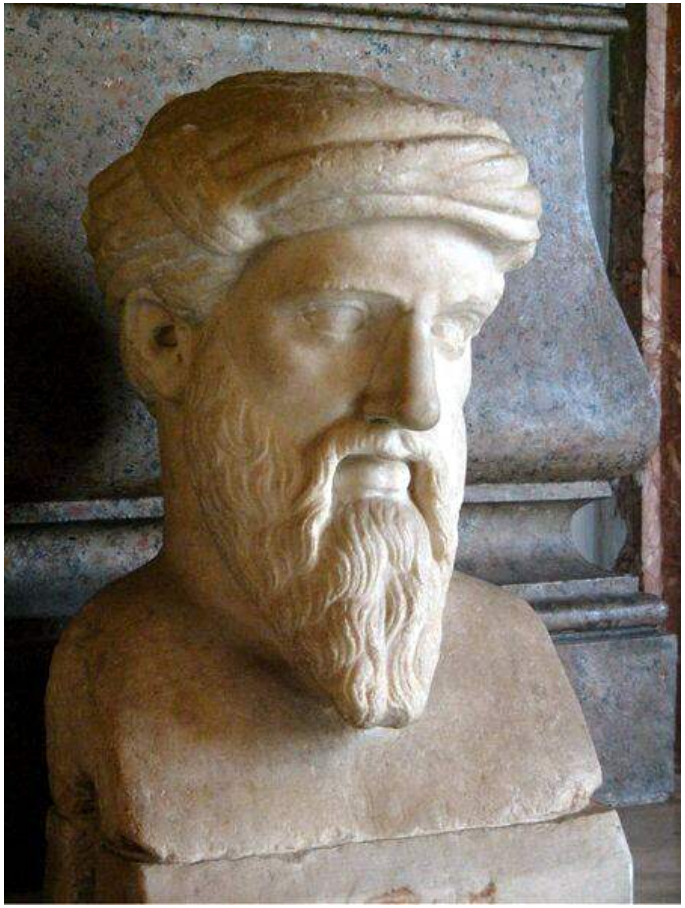
It means that

$$c^2 = a^2 + b^2$$

Pythagoras

" There is geometry in the humming of the strings. "

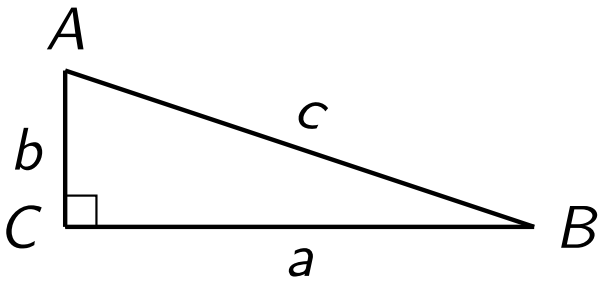
Pythagoras



Pythagoras theorem

The Greek mathematician Pythagoras discovered that :

- in a right angled triangle
- the square of the hypotenuse is equal to the sum of the squares of the other two sides.



Note that the hypotenuse is *the longest side* (c here).

$$a^2 + b^2 = c^2$$

Pythagoras theorem

The Greek mathematician Pythagoras discovered that :

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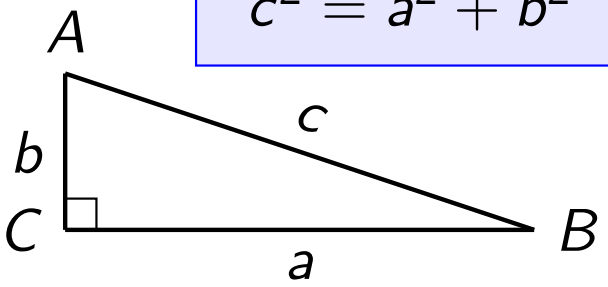
- the ... ne squa

Pythagoras theorem :
if c it the hypotenuse in a right triangle then
$c^2 = a^2 + b^2$



Note that the hypotenuse is *the longest side* (c here).

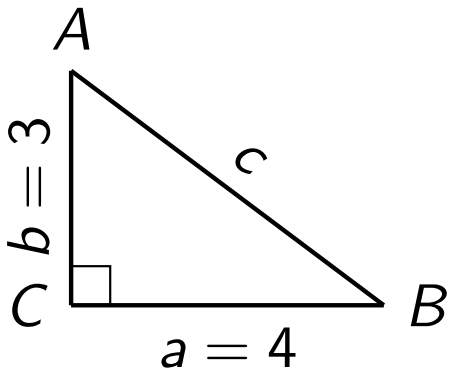
$$a^2 + b^2 = c^2$$

Consequence

If you know two sides of a right triangle, you can calculate the last one !

What is c ?
ABC is rectangle. The hypothenuse is c, and $a = 4, b = 3$. Pythagoras theorem :
$a^2 + b^2 = c^2$

$$\begin{aligned} c^2 &= 4^2 + 3^2 \\ &= 16 + 9 \\ &= 25 \end{aligned}$$



hence

$$c = \sqrt{25} = 5$$

Consequence

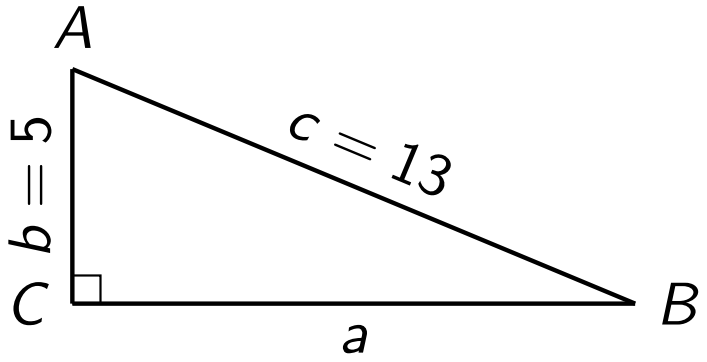
If you know two sides of a right triangle, you can calculate the last one !

What is a ?
ABC is rectangle and $b = 5, c = 13$. The hypotenuse is c : $a^2 + b^2 = c^2$
We rearrange the terms :

$$a^2 = c^2 - b^2$$

and

$$\begin{aligned} a^2 &= 13^2 - 5^2 \\ &= 169 - 25 \\ &= 144 \end{aligned}$$

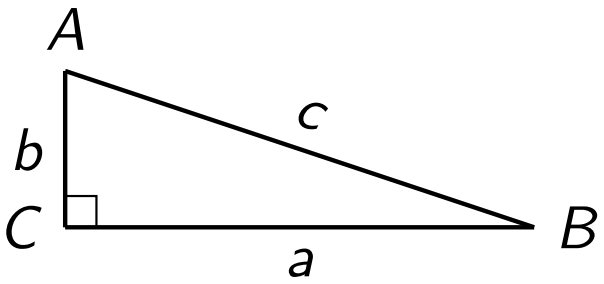


and hence,

$$a = \sqrt{144} = 12$$

Trigonometric functions

Trigonometric functions are functions that **depend** on an angle.



- $\sin \widehat{A} = \dfrac{a}{c} = \dfrac{\text{opposite}}{\text{hypothenuse}}$
- $\cos \widehat{A} = \dfrac{b}{c} = \dfrac{\text{adjacent}}{\text{hypothenuse}}$
- $\tan \widehat{A} = \dfrac{a}{b} = \dfrac{\text{adjacent}}{\text{hypothenuse}} = \dfrac{\sin \widehat{A}}{\cos \widehat{A}}$

Note that Sin is short for Sine, Cos is short for Cosine and Tan is short for Tangent

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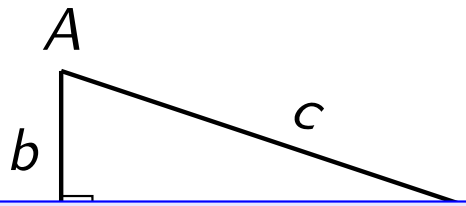

These ratios allow both to

$\rightsquigarrow$ define the trigonometric functions such as Sine

$\rightsquigarrow$ calculate the angle

- $\cos \widehat{A} = \dfrac{b}{c} = \dfrac{\text{adjacent}}{\text{hypothenuse}}$
- $\tan \widehat{A} = \dfrac{a}{b} = \dfrac{\text{adjacent}}{\text{hypothenuse}} = \dfrac{\sin \widehat{A}}{\cos \widehat{A}}$

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Why is it important ? 1/2

Trigonometry helps us find

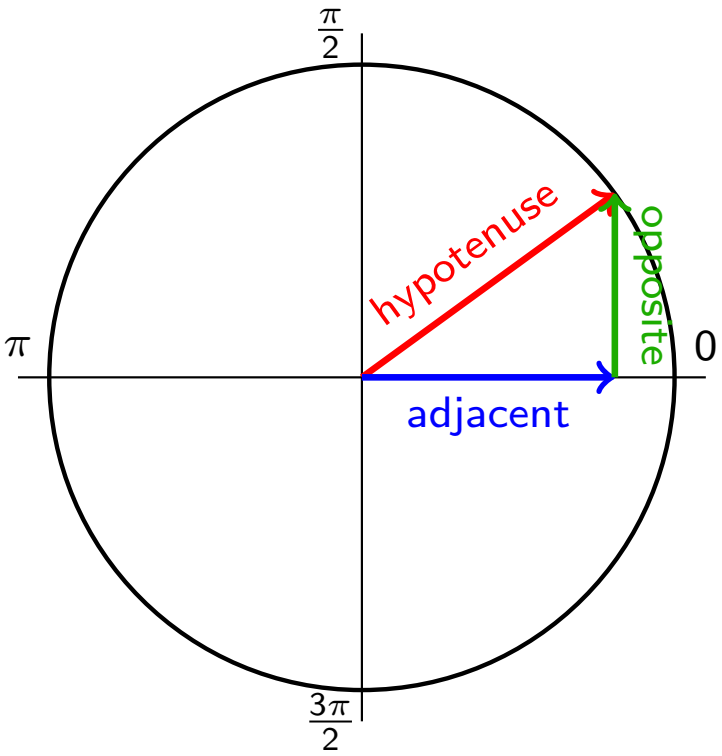
- angles
- distances

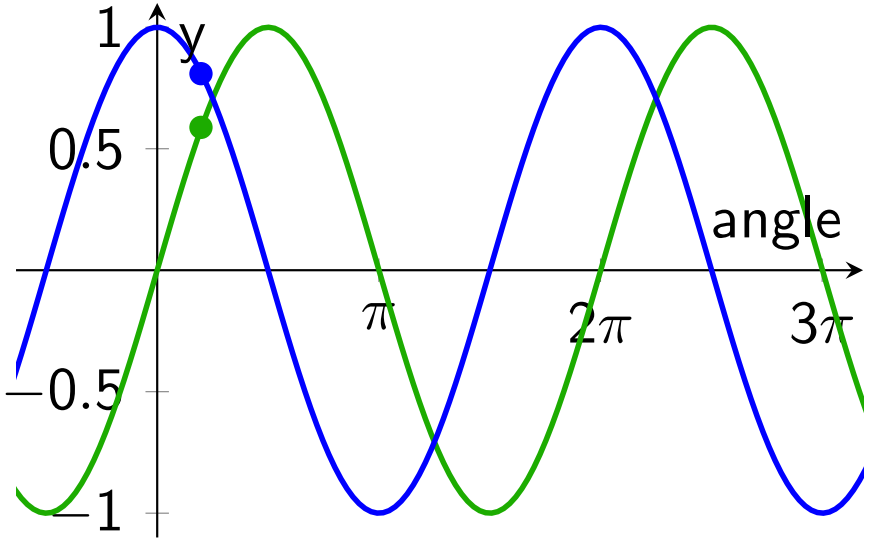
This is useful for

- science (astrophysics)
- engineering (engines)
- video games (mirrors, depth)
- and more !

Why is it important ? 2/2

the function sine and cosine are periodic. Why ? Because they are related to a circle





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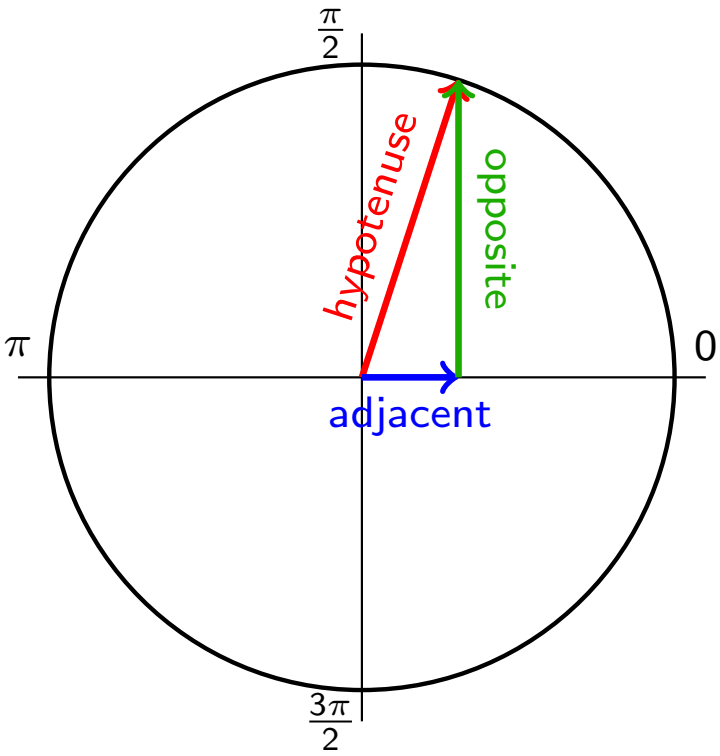

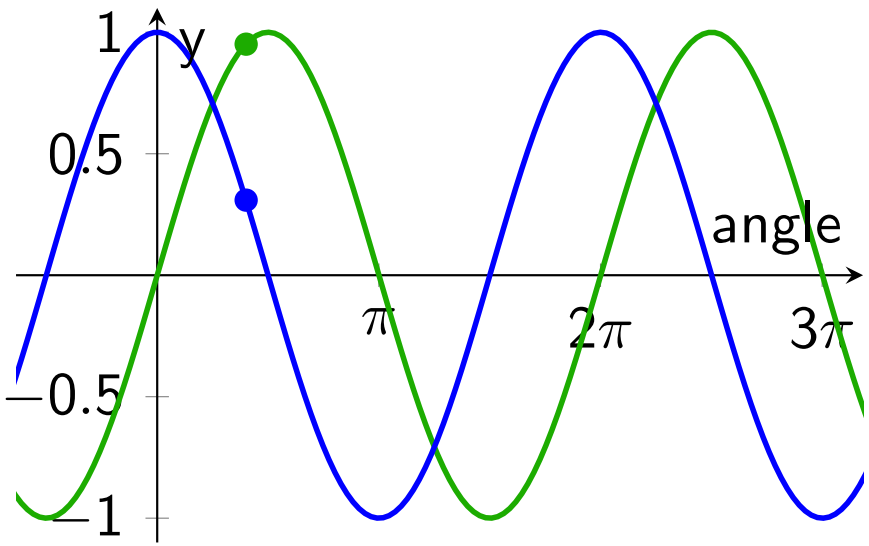

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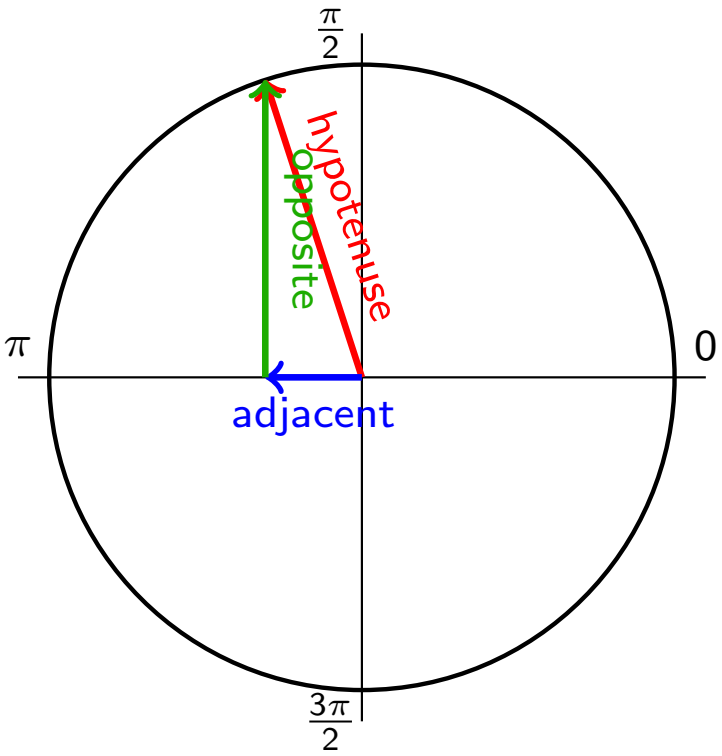

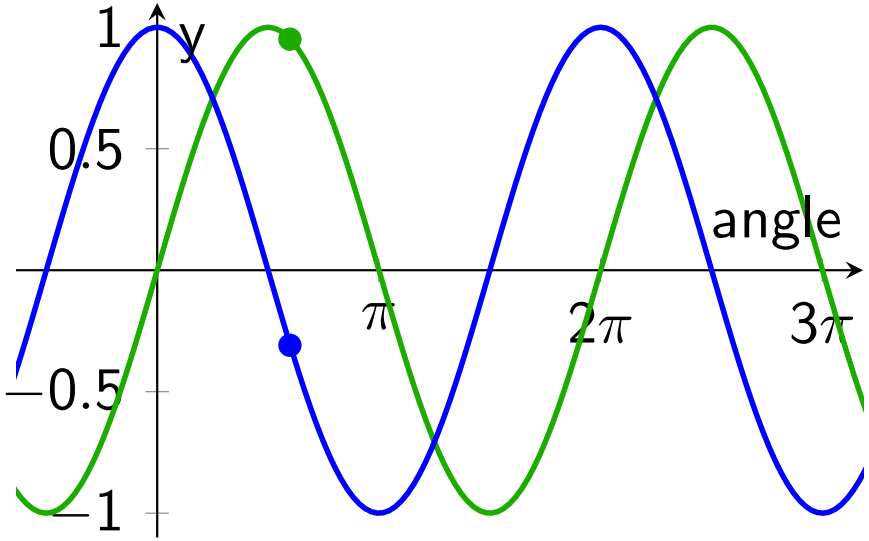

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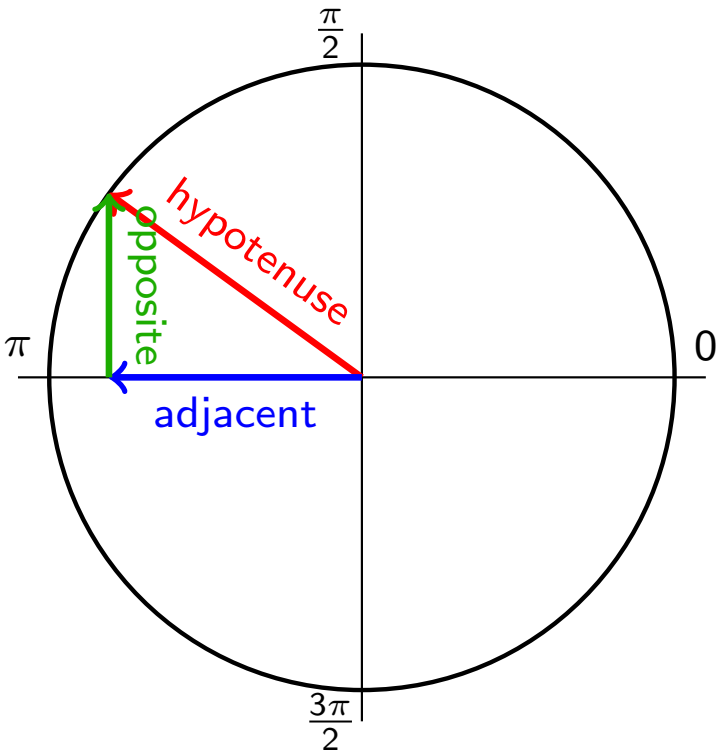

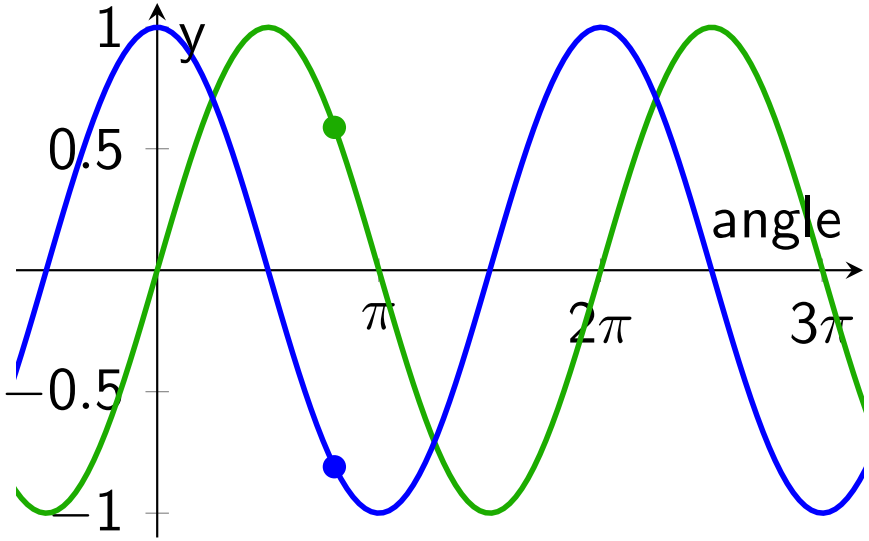

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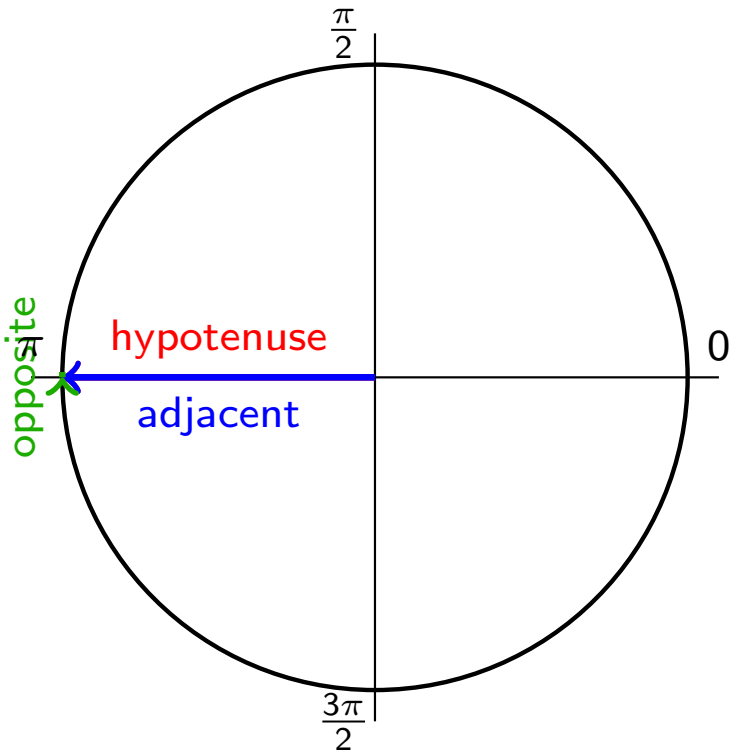

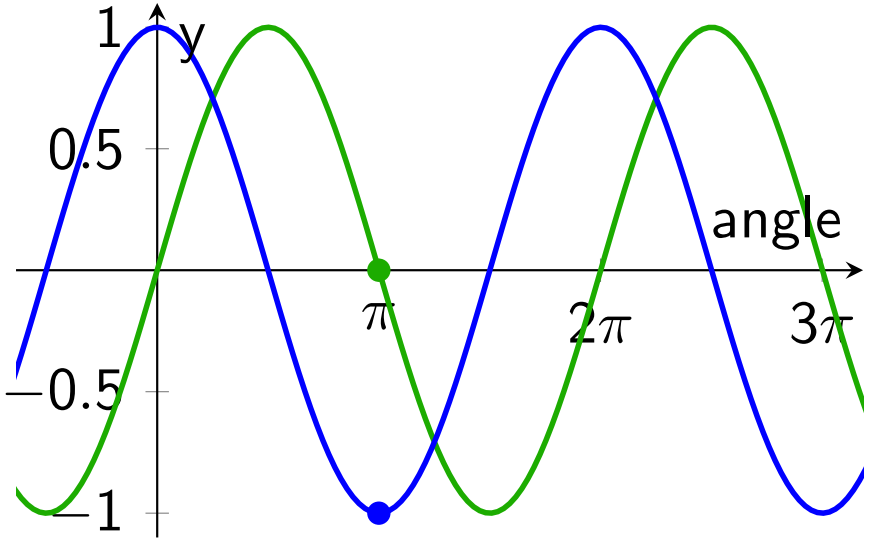

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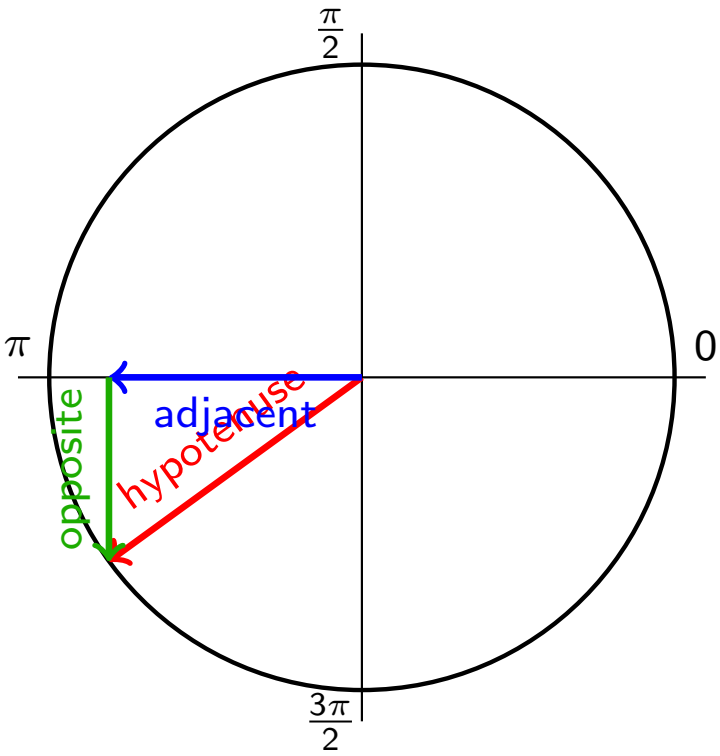

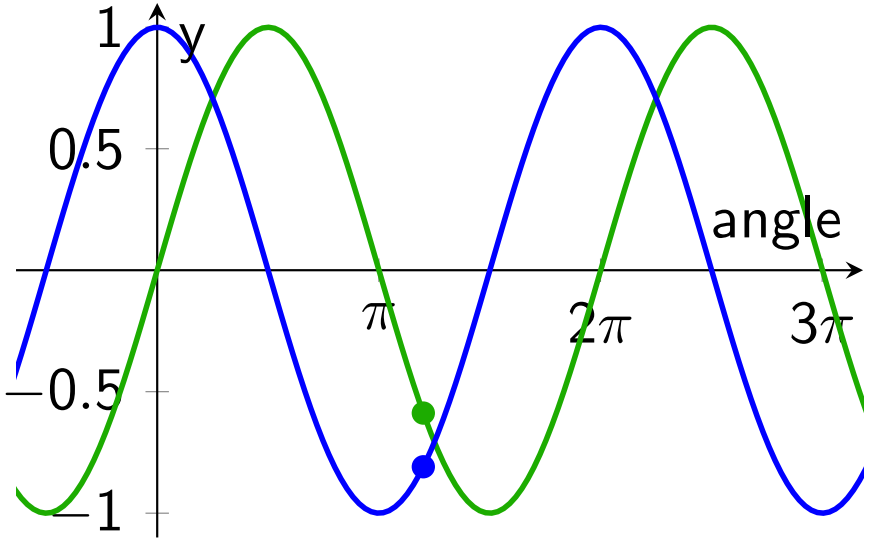

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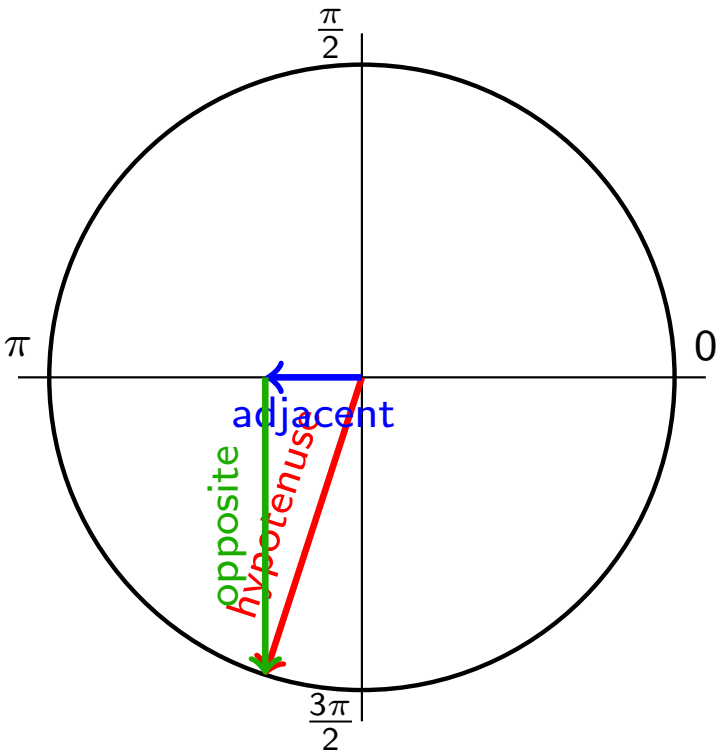

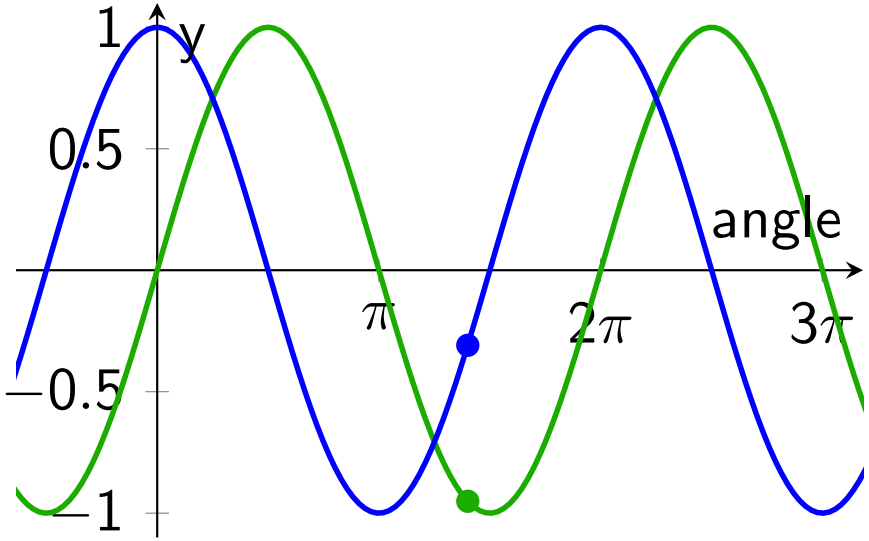

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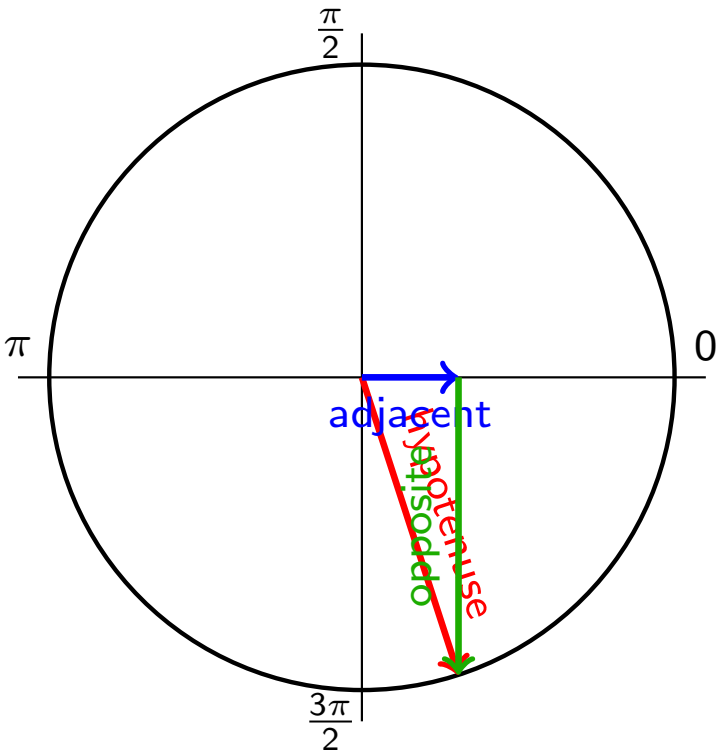

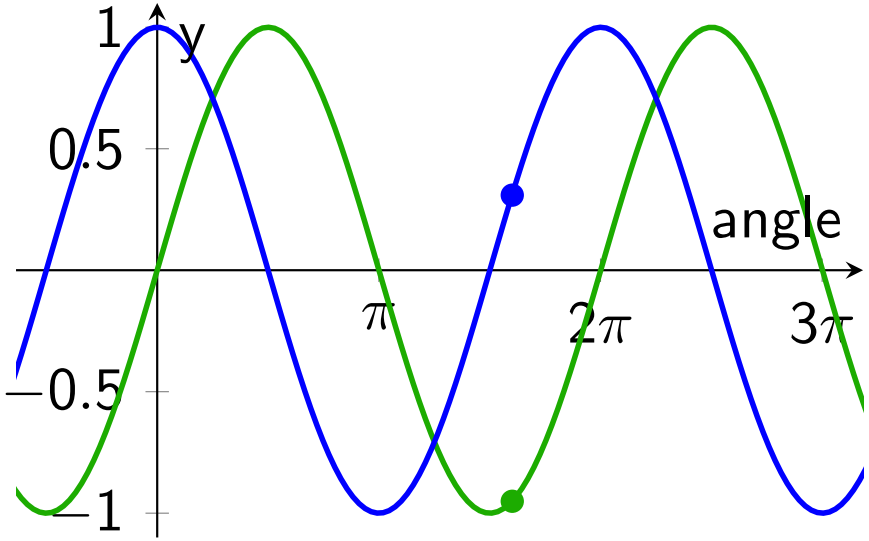

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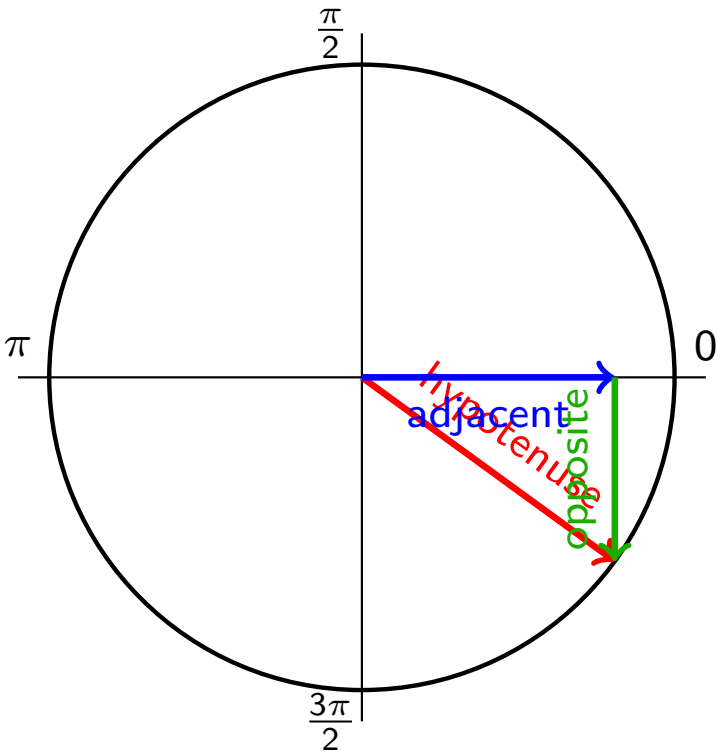

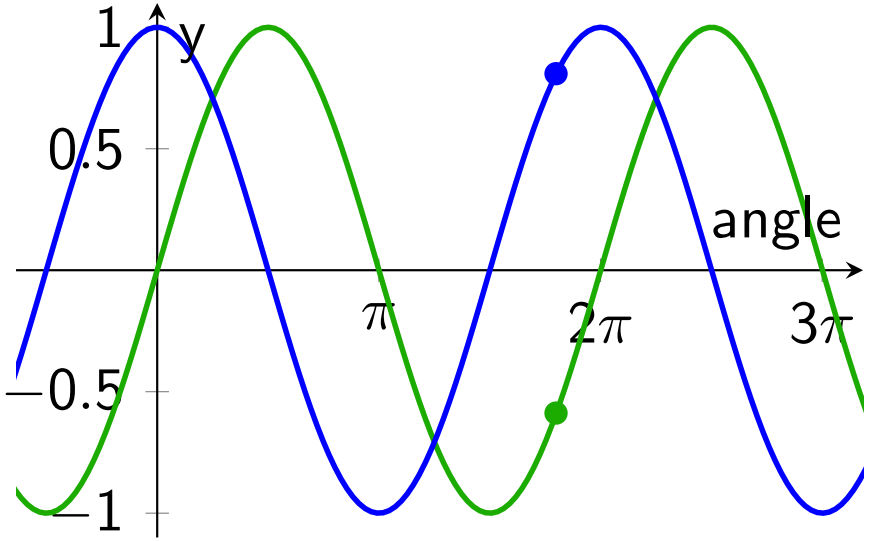

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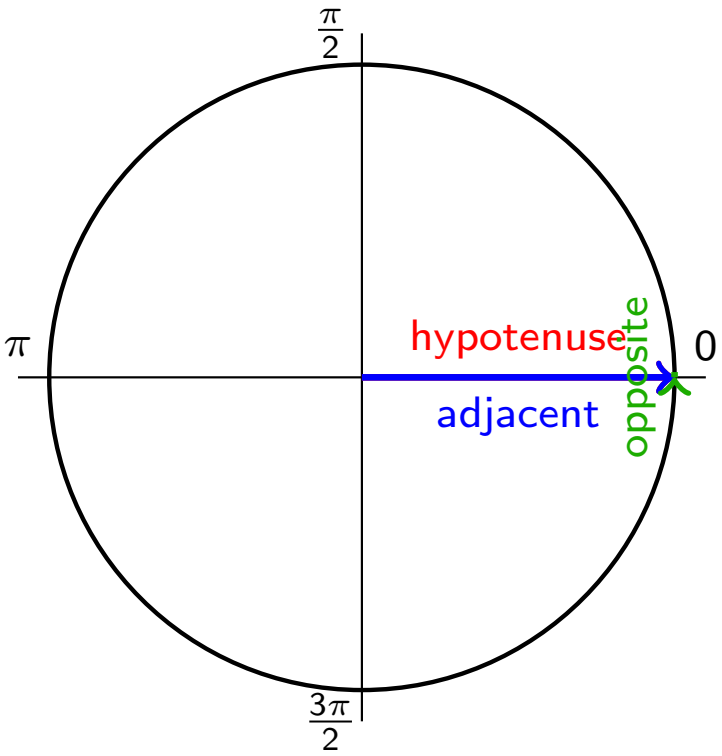

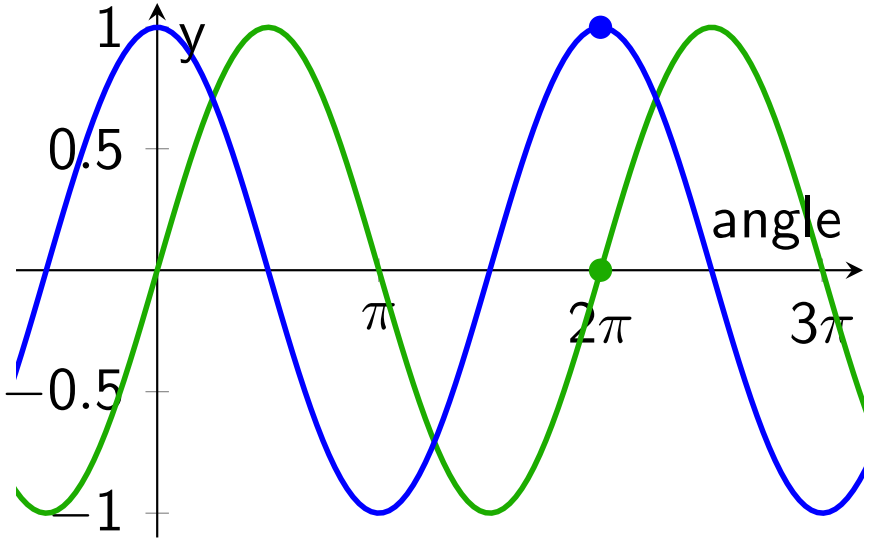

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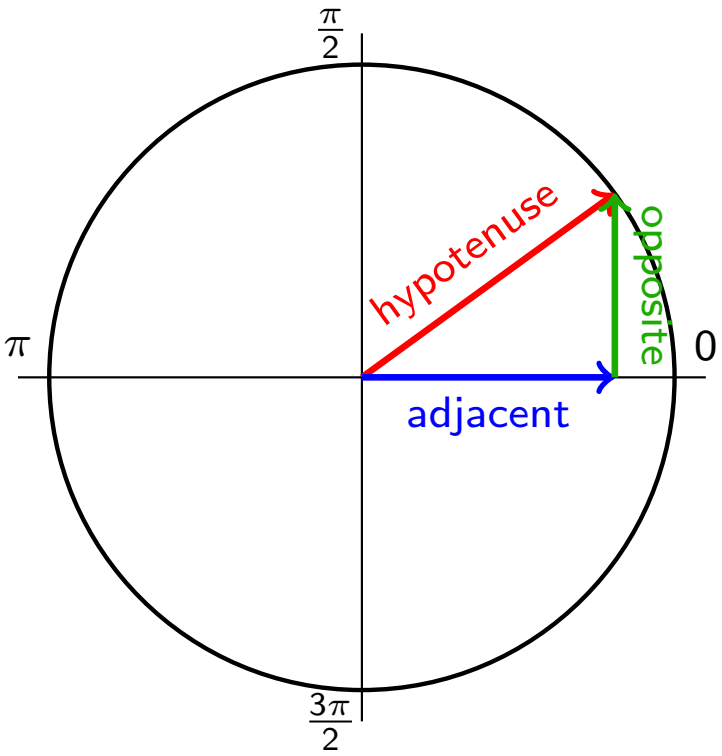

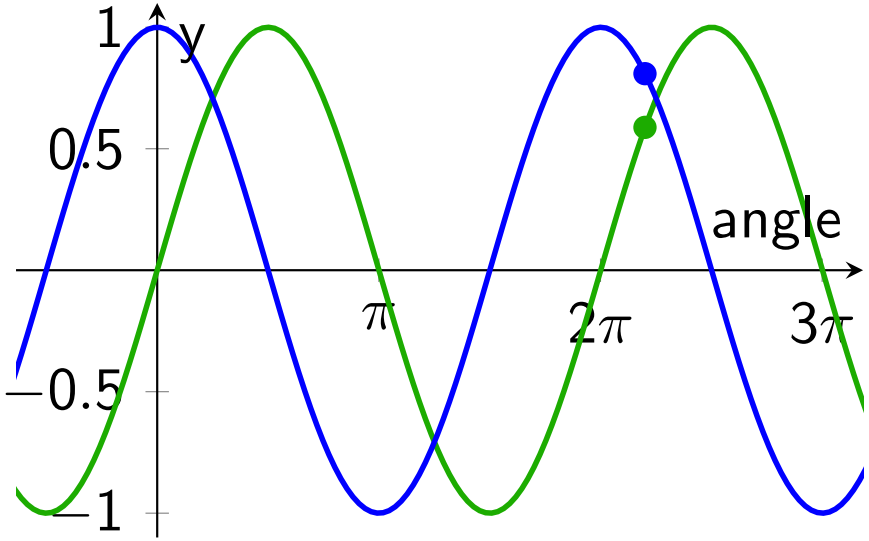

Meaning ?

If we want to calculate the function for an angle larger than a full rotation (i.e., larger than360^o or 2π radians) :
Subtract *as many* full rotations as needed to bring it back between 0° and 360° (or 0 and 2π radians)

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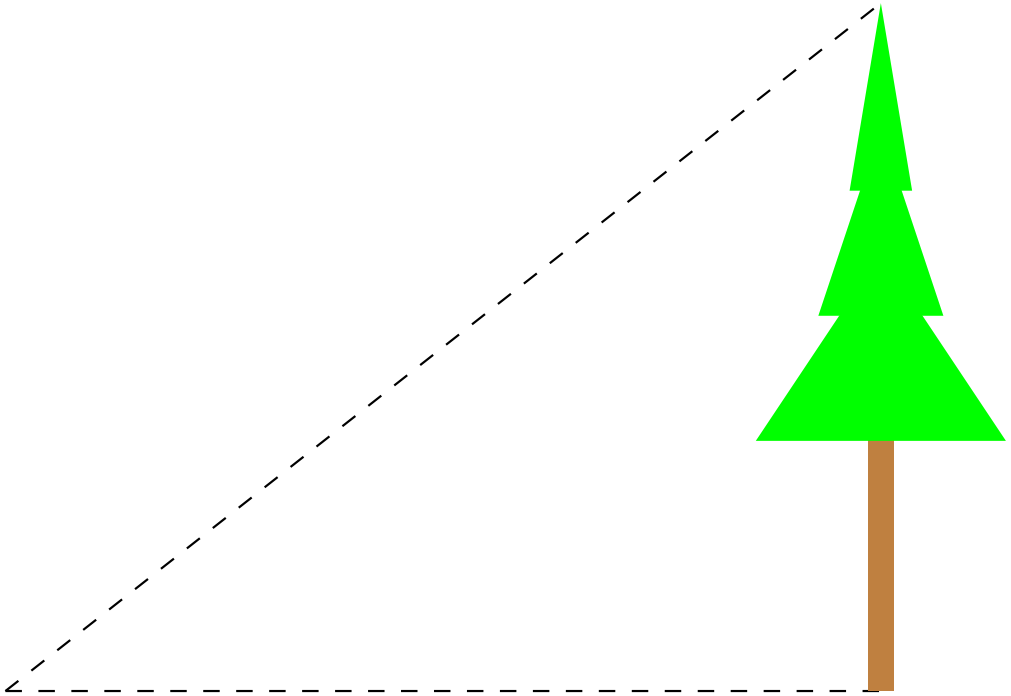
Subtract *as many* full rotations as needed to bring it back between 0° and 360° (or 0 and 2π radians)

370°	10°	2.4πrad	0.4πrad
-20°	340°	-0.5πrad	1.5πrad
800°	80°	4.8πrad	0.8πrad

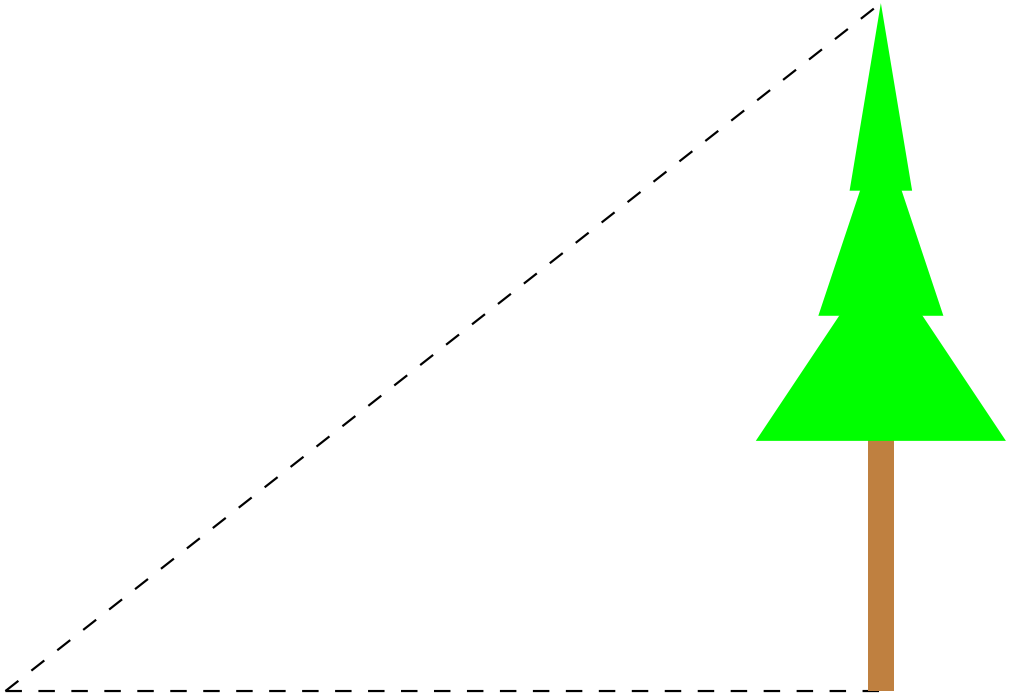
Percentage

A way of understand the *sine* is to see it as the *height* of a tree (or anything), as a percentage, or fraction, of the hypotenuse :



Percentage

A way of understand the *cosine* is to see it as the *distance* to a tree (or anything), as a percentage, or fraction, of the hypotenuse :



A trick to remember the formulas

They can be abbreviated to

- $s = \frac{o}{h}$
- $c = \frac{a}{h}$
- $s = \frac{o}{a}$

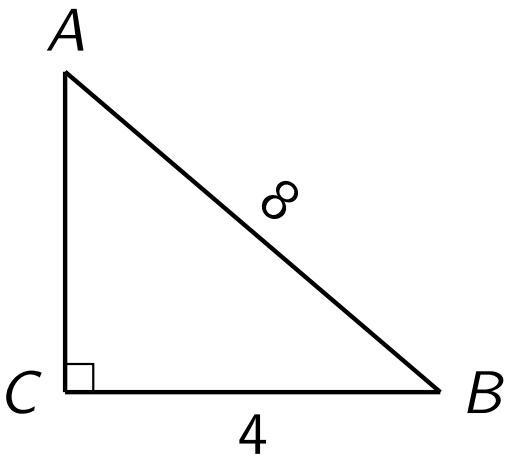
it can be remembered by the word $SOHCAHTOA = S^O{}_H C^A{}_H T^O{}_A$. If it is too weird, some remember a mnemonic sentence such as "Some Old Houses Creak And Howl Through Old Age".

Example : angles

What is the angle $\widehat{A}$?

Which one of the 3 ratios to use ?

- Sine
- Cosine
- Tangent



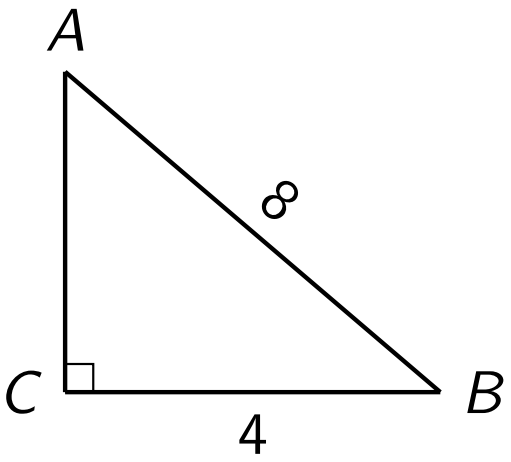
Example : angles

What is the angle $\widehat{A}$?

In relation to A :

- the 4 is the opposite side
- the 8 is the hypotenuse

$\rightsquigarrow$ Opposite and hypotenuse means sin.



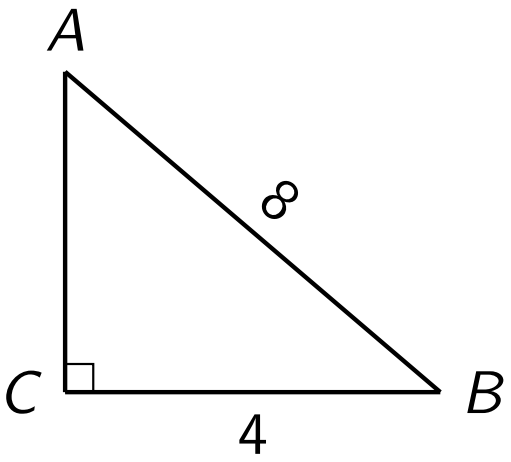
Example : angles

What is the angle $\widehat{A}$?

$$\sin \widehat{A} = \frac{\text{opposite}}{\text{hypothenuse}} = \frac{4}{8} = 0.5$$

So $\sin \widehat{A} = 0.5$.
How can we calculate $\widehat{A}$? $\rightsquigarrow$ *calculator*.

$$\widehat{A} = \text{Inv Sin}(0.5) = 30^o$$

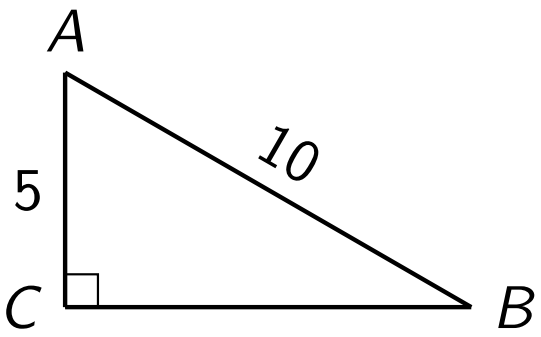


Example : angles

What is the angle $\widehat{A}$?

Which one of the 3 ratios to use ?

- Sine
- Cosine
- Tangent



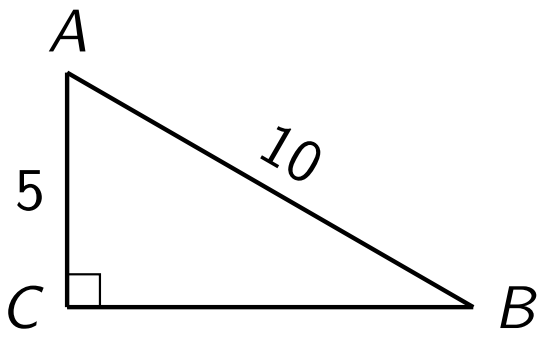
Example : angles

What is the angle $\widehat{A}$?

In relation to A :

- the 5 is the adjacent side
- the 10 is the hypotenuse

$\rightsquigarrow$ adjacent and hypotenuse means cos.



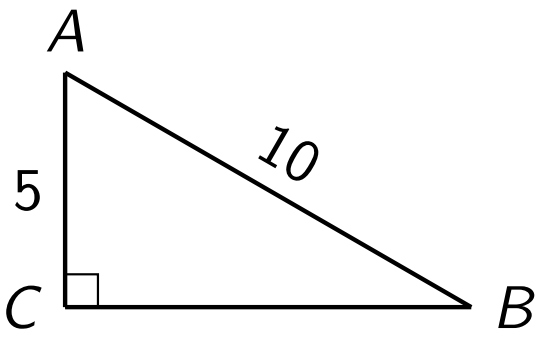
Example : angles

What is the angle $\widehat{A}$?

$$\cos \widehat{A} = \frac{\text{adjacent}}{\text{hypothenuse}} = \frac{5}{10} = 0.5$$

So $\cos \widehat{A} = 0.5$.

$$\widehat{A} = \cos^{-1}(0.5) = 60^o$$

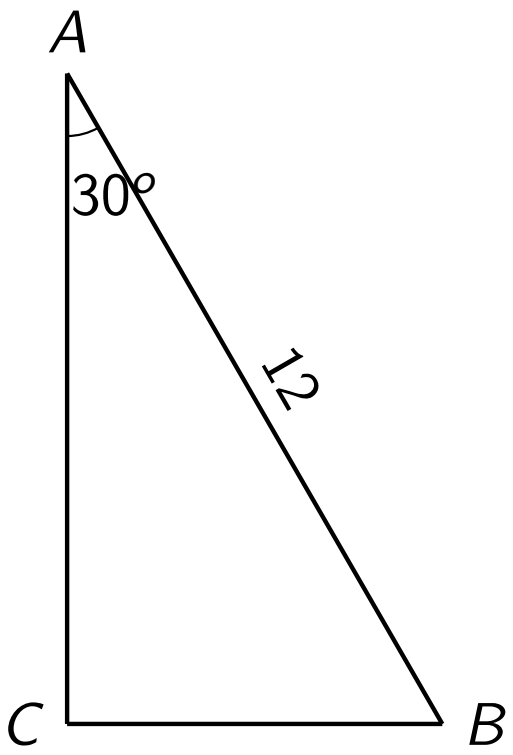


Example : side

What is the side AC ?

Which one of the 3 ratios to use ?

- Sine
- Cosine
- Tangent



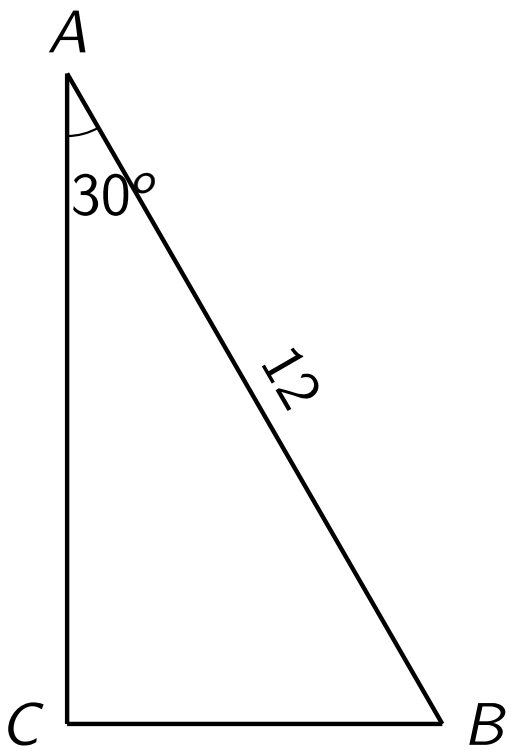
Example : side

What is the side AC ?

we have or want :

- the angle ($\widehat{A} = 30^0$)
- the hypotenuse ($AB = 12$)
- the adjacent side (AC)

Adjacent and hypotenuse $\rightsquigarrow$ cos.



Example : side

What is the side AC ?

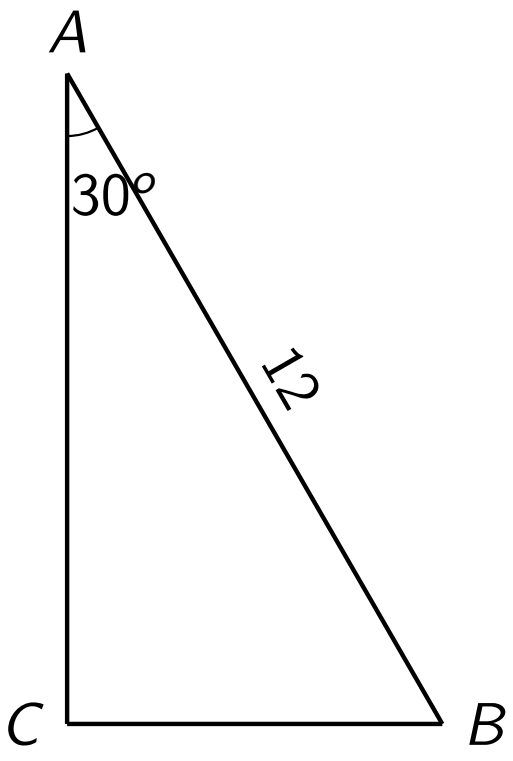
$$\cos \widehat{A} = \frac{adjacent}{hypothenuse} = \frac{AC}{AB}$$

and :

$$AC = \cos \widehat{A} \times AB$$

it follows

$$\begin{aligned} AC &= \cos \widehat{A} \times AB \\ &= \cos(30^o) \times 12 \\ &= 0.866 \times 12 \\ &= 10.39 \end{aligned}$$

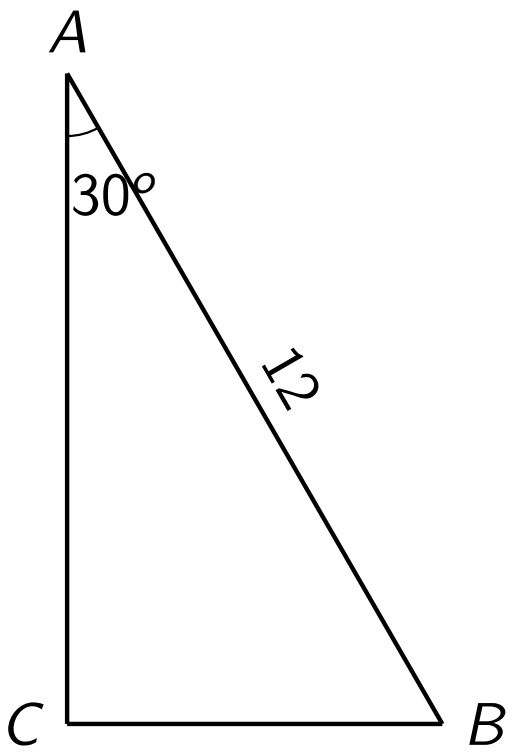


Example : side

What if we want BC ?

Which one of the 3 ratios to use ?

- Sine
- Cosine
- Tangent



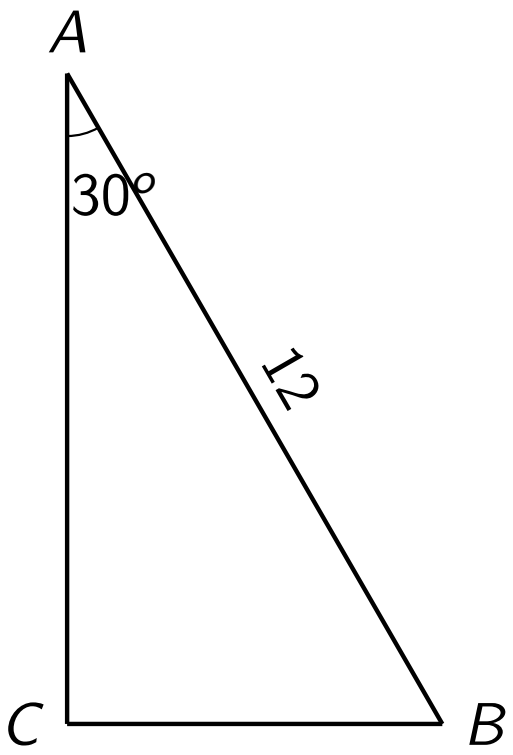
Example : side

What if we want BC ?

we have or want :

- the angle ($\widehat{A} = 30^0$)
- the hypotenuse ($AB = 12$)
- the opposite side (BC)

Opposite and hypotenuse $\rightsquigarrow$ sin.



Example : side

What if we want BC ?

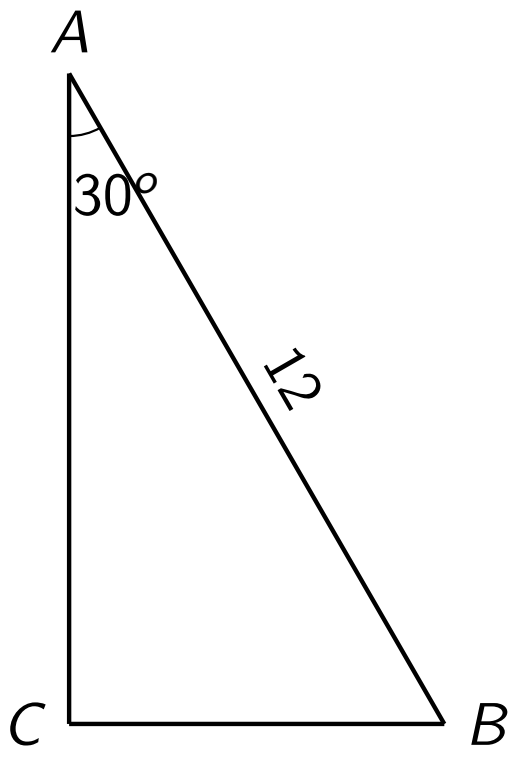
$$\sin \widehat{A} = \frac{opposite}{hypothenuse} = \frac{BC}{AB}$$

and :

$$BC = \sin \widehat{A} \times AB$$

it follows

$$\begin{aligned} BC &= \sin \widehat{A} \times AB \\ &= \sin(30^o) \times 12 \\ &= 0.5 \times 12 \\ &= 6 \end{aligned}$$



How to cut a pizza in six ?

It is easy to cut a pizza in 4 or in 8.

How to cut a pizza in six ?

It is easy to cut a pizza in 4 or in 8.
But, in 6 ?

photo by M. Panades

6 slices

Six slices means that each slice is associated with an angle of :

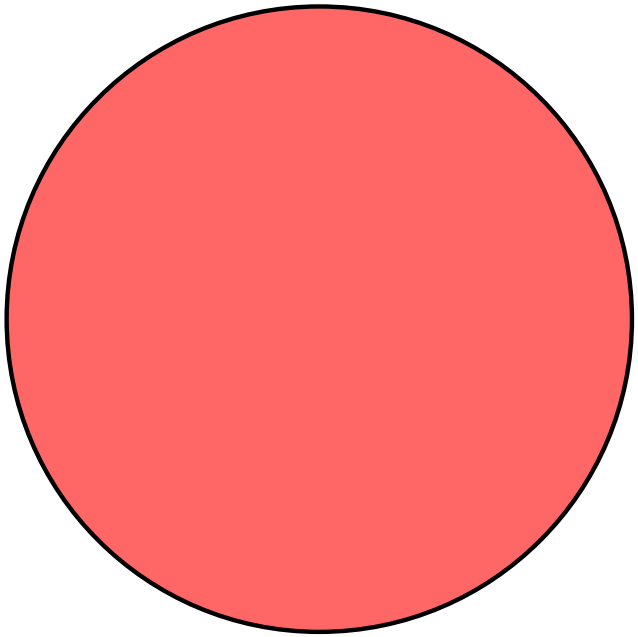
6 slices

Six slices means that each slice is associated with an angle of :

$$\frac{2\pi}{6} = \frac{\pi}{3} rad. \text{ or } \frac{360}{6} = 60^o$$

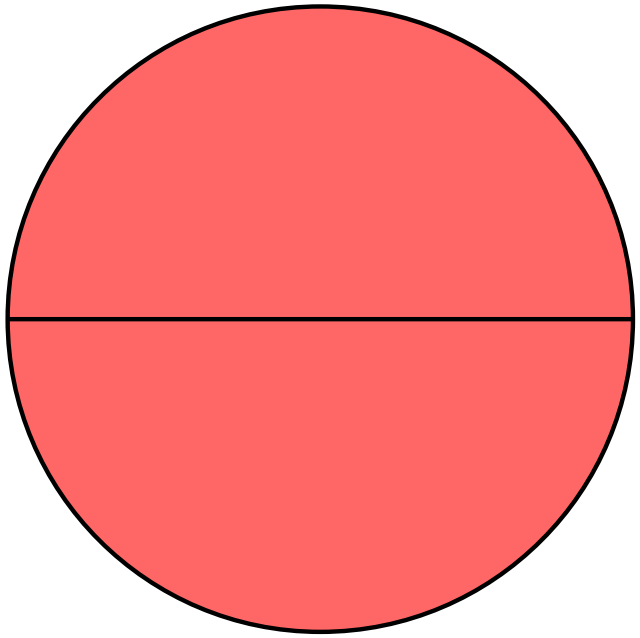
or, $\cos\frac{\pi}{3} = 0.5$! And that tells us how to divide our pizza !

Cuts



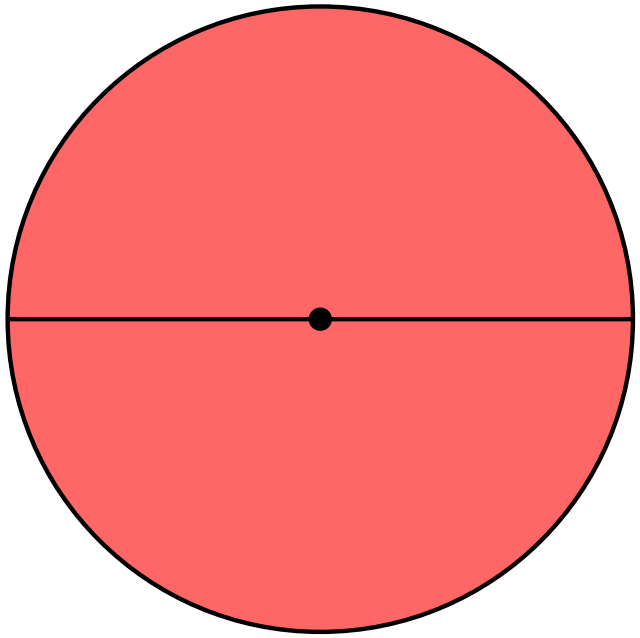
Serve the pizza !

Cuts



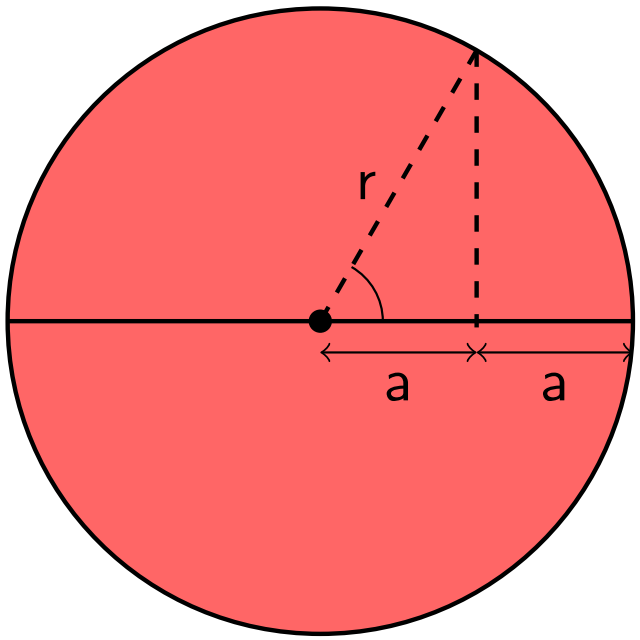
Cut it in half.

Cuts



Note the center.

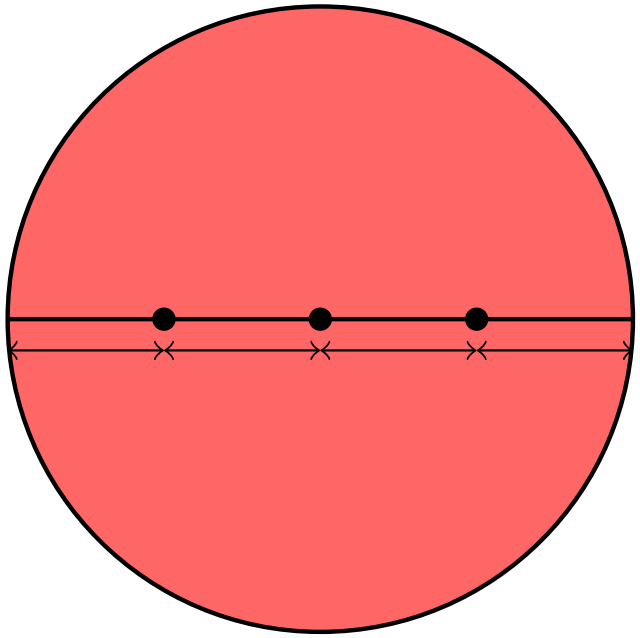
Cuts



Here is the math. The slice has an angle of 60°, and hence

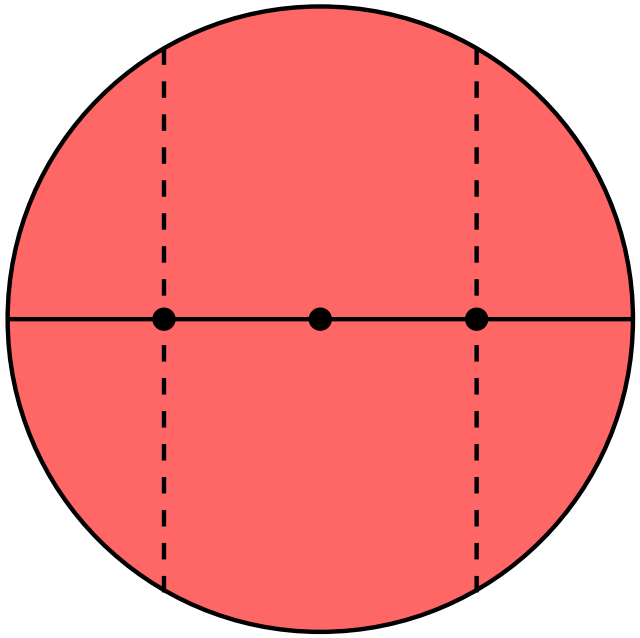
$$a = \cos(60) \times r = \frac{1}{2} r$$

Cuts



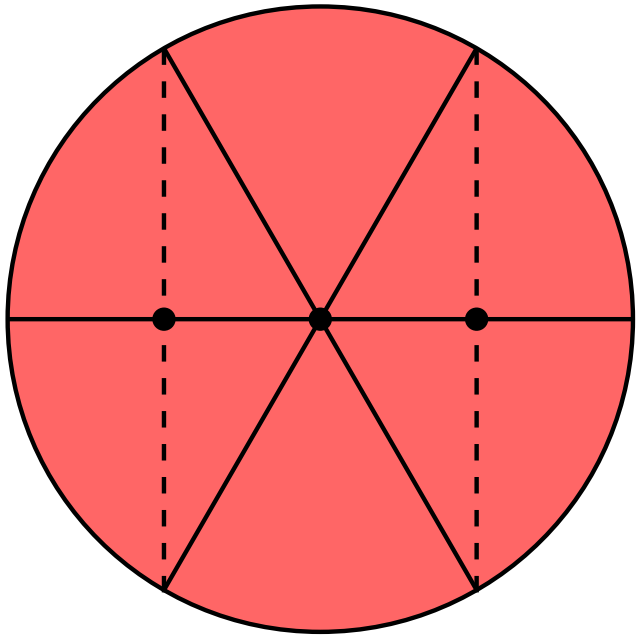
Note the quarters.

Cuts



Go on the verticals.

Cuts



Cut and share !