Mathematics for Engineers I
Introduction to linear algebra : Matrices

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What will you learn

These lessons will be mostly, obviously, maths :

- what is *matrices*
- how it works - practice !
- why it is useful - physics and engineering

I will try to explain in detail (maybe too much). *Please* let me know if you do not understand something.

What you will have to remember

- what is a matrix
- matrix algebra
  - addition, substraction
  - multiplication
- determinent and minors for
  - 2×2 matrices
  - 3×3 matrices
- solving linear systems with matrices

What is 1

In algebra, 1 is the number so

$$a \times 1 = 1 \times a = a$$

is there an equivalent of 1 for the matrices ?

What is 1

So we want a matrix I with, for any matrix A :

$$A \times I = I \times A = A$$

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, and $I = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$, then :

$$A \times I = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix} = A$$

What is 1

It means :

$$\begin{cases} ae+bg & = & a \\ af+bh & = & b \\ ce+dg & = & c \\ cf+dh & = & d \end{cases}$$

any idea of what should be the coefficients e, f, g and h ?

What is 1

The only general solution is a matrix with

- 1 on the diagonal
- 0 everywhere else

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ or, } I = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 \end{pmatrix}.$$

I is called the identity matrix.

Division

We have seen how to multiply two matrices $A \times B$, or one scalar and a matrix, $c \times A$. Can we divide by a matrix ? Something like c/A, A/B or $\frac{A}{B}$?

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No, we cannot !

Instead of dividing, we can multiply by the inverse.

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It is the same thing as saying $4 \div 5 = 4 \times \frac{1}{5}$.

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But be careful ! In the matrix world,

$$4 \times \frac{1}{5} \neq \frac{1}{5} \times 4$$

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What is an inverse matrix ?

The inverse of M, N, is defined by :

- $N \times M = I$
- $M \times N = I$

What is an inverse matrix ?

The inverse of M, N, is defined by :

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- $M \times N = I$

N is then the inverse of M and is noted M^{-1}.

Let's find it for a 2×2 matrix – start

So let's have the matrix M :

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

and suppose that M^{-1} is :

$$M^{-1} = \begin{pmatrix} e & f \\ g & h \end{pmatrix},$$

Let's find it for a 2×2 matrix – $M \times M^{-1}$

Because we have $M \times M^{-1} = I$, it means that :

$$\begin{cases} a \times e + b \times g = 1 \\ a \times f + b \times h = 0 \\ c \times e + d \times g = 0 \\ c \times f + d \times h = 1 \end{cases}$$

Let's find it for a 2×2 matrix – $M \times M^{-1}$

Because we have $M \times M^{-1} = I$, it means that :

$$\begin{cases} a \times e + b \times g = 1 \\ a \times f + b \times h = 0 \\ c \times e + d \times g = 0 \\ c \times f + d \times h = 1 \end{cases}$$

It does not seem easy to solve.

Let's find it for a 2×2 matrix – a trick

Let's look at matrix P :

$$P = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix},$$

And let's calculate $Q = M \times P$.
Its first element is :

$$\begin{aligned} q_{11} &= m_{11} \times p_{11} + m_{12} \times p_{21} \\ &= a \times \left(d \frac{1}{ad - bc} \right) + b \times \left(-c \frac{1}{ad - bc} \right) \\ &= \frac{ad}{ad - bc} + \frac{-bc}{ad - bc} \\ &= 1 \end{aligned}$$

Let's find it for a 2×2 matrix – a trick

Let's look at matrix P :

$$P = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix},$$

And let's calculate $Q = M \times P$.
Its second element is :

$$\begin{aligned} q_{12} &= m_{11} \times p_{12} + m_{12} \times p_{22} \\ &= a \times \left(-c \frac{1}{ad - bc} \right) + c \times \left(a \frac{1}{ad - bc} \right) \\ &= \frac{-ac}{ad - bc} + \frac{ac}{ad - bc} \\ &= 0 \end{aligned}$$

Let's find it for a 2×2 matrix – a trick

Let's look at matrix P :

$$P = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix},$$

And let's calculate $Q = M \times P$.
Its third element is :

$$\begin{aligned} q_{21} &= m_{21} \times p_{11} + m_{22} \times p_{21} \\ &= c \times \left(d \frac{1}{ad - bc} \right) + d \times \left(-c \frac{1}{ad - bc} \right) \\ &= \frac{-cd}{ad - bc} + \frac{cd}{ad - bc} \\ &= 0 \end{aligned}$$

Let's find it for a 2×2 matrix – a trick

Let's look at matrix P :

$$P = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix},$$

And let's calculate $Q = M \times P$.
And finally the last element is :

$$\begin{aligned} q_{22} &= m_{21} \times p_{12} + m_{22} \times p_{22} \\ &= c \times \left(-b\frac{1}{ad - bc}\right) + d \times \left(a\frac{1}{ad - bc}\right) \\ &= \frac{-bc}{ad - bc} + \frac{ad}{ad - bc} \\ &= 1 \end{aligned}$$

Let's find it for a 2×2 matrix – the trick

It means that

$$Q = M \times P = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Let's find it for a 2×2 matrix – the trick

And let's calculate $Q = P \times M$. $P = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Its first element is :

$$
\begin{aligned}
q_{11} &= p_{11} \times m_{11} + p_{12} \times m_{21} \\
&= \left(d \frac{1}{ad - bc} \right) \times a + \left(-b \frac{1}{ad - bc} \right) \times c \\
&= \frac{ad}{ad - bc} + \frac{-bc}{ad - bc} \\
&= 1
\end{aligned}
$$

Let's find it for a 2×2 matrix – the trick

And let's calculate $Q = P \times M$. $P = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

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\begin{aligned}
q_{12} &= p_{11} \times m_{12} + p_{12} \times m_{22} \\
&= \left(d\frac{1}{ad - bc}\right) \times b + \left(-b\frac{1}{ad - bc}\right) \times d \\
&= \frac{bd}{ad - bc} + \frac{ac}{-bd} \\
&= 0
\end{aligned}
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Let's find it for a 2×2 matrix – the trick

And let's calculate $Q = P \times M$. $P = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

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\begin{aligned}
q_{21} &= p_{21} \times m_{11} + p_{22} \times m_{21} \\
&= \left(-c\frac{1}{ad-bc}\right) \times a + \left(a\frac{1}{ad-bc}\right) c \\
&= \frac{-ca}{ad-bc} + \frac{ca}{ad-bc} \\
&= 0
\end{aligned}
$$

Let's find it for a 2×2 matrix – the trick

And let's calculate $Q = P \times M$. $P = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

And finally the last element is :

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q_{22} &= p_{21} \times m_{12} + p_{22} \times m_{22} \\
&= \left(-c\frac{1}{ad-bc}\right) \times b + \left(a\frac{1}{ad-bc}\right) \times d \\
&= \frac{-bc}{ad-bc} + \frac{ad}{ad-bc} \\
&= 1
\end{aligned}
$$

Let's find it for a 2×2 matrix – the end ?

So we have :

- $P \times M = I$
- $M \times P = I$

It means that $P = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, is the inverse of M and hence, $P = M^{-1}$. Is it the end ?

Let's find it for a 2×2 matrix – the end ?

So we have :

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It means that $P = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, is the inverse of M and hence, $P = M^{-1}$. Is it the end ?

What happens if $ad - bc = 0$?

A special number

Because of $\dfrac{1}{ad-bc}$, it means that $ad - bc \neq 0$. If it is zero, it means that the *inverse of matrix M does not exist*.
This is why it has a special name : the determinant of the matrix M.

The determinant

The determinant of a matrix M is noted as :

- $\det M$
- $|M|$
- $\det(M)$

One of the major properties of the determinant is :

If $\det M \neq 0$ *then* M^{-1} *exists.*

Matrix 2×2

For $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $\det M = ad - bc$.

If $A = \begin{pmatrix} 2 & 5 \\ 1 & 4 \end{pmatrix}$ then

$$\det A = 2 \times 4 - 1 \times 5 = 3 \neq 0$$

A can be inverted !

Matrix 2×2

For $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $\det M = ad - bc$.

If $B = \begin{pmatrix} 1 & 2 \\ 6 & 3 \end{pmatrix}$ then

$$\det B = 1 \times 3 - 2 \times 6 \neq 0$$

B can be inverted !

Matrix 2×2

For $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $\det M = ad - bc$.

If $C = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}$ then

$$\det B = 1 \times 6 - 2 \times 3 = 0$$

C can not be inverted.

Easy inverse of a 2×2 matrix

If

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

then, if $\det A \neq 0$, the inverse of A is found by

$$A^{-1} = \frac{1}{\det A} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix},$$

Easy inverse of a 2×2 matrix

Or :

- Write A
- exchange the a and the d
- change the sign of the b and the c
- multiply by one over the determinant $\det A$

A few illustrations

If

$$A = \begin{pmatrix} 2 & 5 \\ 1 & 4 \end{pmatrix},$$

then

$$\det A = 2 \times 4 - 1 \times 5 = 8 - 5 = 3$$

$\det A \neq 0$ it means that A has an inverse ! Let's calculate A^{-1} :

$$A^{-1} = \frac{1}{3} \begin{pmatrix} 4 & -5 \\ -1 & 2 \end{pmatrix}$$

A few illustrations

If

$$B = \begin{pmatrix} 1 & 2 \\ 6 & 3 \end{pmatrix},$$

then

$$\det B = 1 \times 3 - 6 \times 2 = 3 - 12 = -9$$

$\det B \neq 0$ it means that B has an inverse !
Let's calculate B^{-1} :

$$B^{-1} = -\frac{1}{9} \begin{pmatrix} 3 & -2 \\ -6 & 1 \end{pmatrix}.$$

3×3 determinant

The rule to calculate the determinant in a 3×3 matrix is
1/
add the product of each right diagonals (from top left to bottom right) :

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = aei + bfg + cdh$$

To complete the diagonal, pick the remaining terms.

3×3 determinant

The rule to calculate the determinant in a 3×3 matrix is
2/
subtract the product of each left diagonals (from top right to bottom left) :

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = -ceg - bdi - afh$$

To complete the diagonal, pick the remaining terms.

3×3 determinant

The rule to calculate the determinant in a 3×3 matrix is
3/
the determinant follows :

$$\det A = aei + bfg + cdh - ceg - bdi - afh$$

To complete the diagonal, pick the remaining terms.

Illustration

For instance, if

$$A = \begin{pmatrix} 1 & 4 & 3 \\ 2 & 3 & 4 \\ 3 & -1 & 2 \end{pmatrix}$$

then the determinant is :

$$\det A = \begin{array}{ll} +1 \times 3 \times 2 & \textit{first right diagonal} \\ +4 \times 4 \times 3 & \textit{second right diagonal} \\ +3 \times 2 \times -1 & \textit{third right diagonal} \\ -3 \times 3 \times 3 & \textit{first left diagonal} \\ -4 \times 2 \times 2 & \textit{second left diagonal} \\ -1 \times 4 \times -1 & \textit{third left diagonal} \end{array}$$

and $\det A = 6 + 48 + (-6) - 27 - 16 + 4 = 9.$

Context

Let's solve equations of the form

$$\begin{cases} ax & +by & = & p \\ cx & +dy & = & q \end{cases}$$

for x and y.

Rewritting

- Let's put the coefficients of x and y in a matrix :$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$
- Let's consider x and y as a *matrix* variable : $X = \begin{pmatrix} x \\ y \end{pmatrix}$
- Let's consider p and q as a matrix : $B = \begin{pmatrix} p \\ q \end{pmatrix}$

Multiplying the matrices $A \times X$ actually gives us B, and hence the original equations !

General form

The equations are now in the general form :

$$A \times X = B$$

Can we divide both sides by the matrix A ?

Of the use of the inverse

We multiply *both* sides by the inverse of A :

$$AX = B \Rightarrow A^{-1}AX = A^{-1}B$$

Of the use of the inverse

We multiply *both* sides by the inverse of A :

$$AX = B \Rightarrow A^{-1}AX = A^{-1}B$$

We can cancel the matrix A on the left hand side !

$$X = A^{-1}B$$

the solution

We have now

$$X = A^{-1}B$$

We read x and y in $X = \begin{pmatrix} x \\ y \end{pmatrix}$ to have the solution.

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We read x and y in $X = \begin{pmatrix} x \\ y \end{pmatrix}$ to have the solution.

It works even for thousands of unknowns.

Illustration 1

Let's solve :

$$\begin{cases} 3x & -2y & = & 6 \\ 2x & +y & = & 11 \end{cases}$$

for x and y.

Re-write the equations in matrix form $AX = B$:

$$\begin{pmatrix} 3 & -2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \end{pmatrix}$$

Illustration 1

Let's solve :

$$\begin{cases} 3x & -2y & = & 6 \\ 2x & +y & = & 11 \end{cases}$$

for x and y.

The determinant of A is :

$$\det A = 3 - (-2 \times 2) = 7$$

A is invertible and A^{-1} is :

$$A^{-1} = \frac{1}{7} \begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix}$$

Illustration 1

Let's solve :

$$\begin{cases} 3x & -2y & = & 6 \\ 2x & +y & = & 11 \end{cases}$$

for x and y.

We just have to calculate $A^{-1}B$ to have the solution :

$$A^{-1}B = \frac{1}{7}\begin{pmatrix} 1 \times 6 + 2 \times 11 = 28 \\ -2 \times 6 + 3 \times 11 = 21 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

Illustration 1

Let's solve :

$$\begin{cases} 3x & -2y & = & 6 \\ 2x & +y & = & 11 \end{cases}$$

for x and y.

by reading x and y in $X = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, we have the solution : $x = 4$ and $y = 3$.

Illustration 1

Let's solve :

$$\begin{cases} 3x & -2y & = & 6 \\ 2x & +y & = & 11 \end{cases}$$

for x and y.

Let's verify now :

- $3x - 2y = 3 \times 4 - 2 \times 3 = 6$
- $2x + y = 2 \times 4 + 3 = 11$

Illustration 2

Let's solve :

$$\begin{cases} 2x & +4y & = & 5 \\ 2x & +y & = & 2 \end{cases}$$

for x and y.
Re-write the equations in matrix form $AX = B$:

$$\begin{pmatrix} 2 & 4 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

Illustration 2

Let's solve :

$$\begin{cases} 2x & +4y & = & 5 \\ 2x & +y & = & 2 \end{cases}$$

for x and y.
The determinant of A is :

$$\det A = 2 \times 1 - 2 \times 4 = -6$$

A is invertible and A^{-1} is :

$$A^{-1} = -\frac{1}{6} \begin{pmatrix} 1 & -4 \\ -2 & 2 \end{pmatrix}$$

Illustration 2

Let's solve :

$$\begin{cases} 2x & +4y & = & 5 \\ 2x & +y & = & 2 \end{cases}$$

for x and y.
We just have to calculate $A^{-1}B$ to have the solution :

$$A^{-1}B = -\frac{1}{6}\begin{pmatrix} 1 \times 5 + (-4) \times 2 = -3 \\ -2 \times 5 + 2 \times 2 = -6 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 1 \end{pmatrix}$$

Illustration 2

Let's solve :

$$\begin{cases} 2x & +4y & = & 5 \\ 2x & +y & = & 2 \end{cases}$$

for x and y.

by reading x and y in $X = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.5 \\ 1 \end{pmatrix}$, we have the solution : $x = 1$ and $y = 0.5$.

Illustration 2

Let's solve :

$$\begin{cases} 2x & +4y & = & 5 \\ 2x & +y & = & 2 \end{cases}$$

for x and y.
Let's verify now :

- $2x + 4y = 2 \times 0.5 + 4 \times 1 = 5$
- $2x + y = 2 \times 0.5 + 1 = 2$

Matrix form

Let's consider a system in the matrix form :

$$A \times X = B$$

with

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, B = \begin{pmatrix} p \\ q \end{pmatrix}$$

note that I used x_1 and x_2 instead of x and y !

Sub Matrices

Let's note A_i the matrix A where the ith column is *replaced* with B.

$$A_1 = \begin{pmatrix} p & b \\ q & d \end{pmatrix}, A_2 = \begin{pmatrix} a & p \\ c & q \end{pmatrix}$$

Cramers rule

Cramers' rule allow to calculate the solution of simultaneous equations with just the use of dets :

- the determinant of the matrix A
- the determinant of each of the matrices A_i

Then the solution is :

$$x_i = \frac{\det A_i}{\det A}$$

The inverse is not calculated !

Illustration 1

Let's solve :

$$\begin{cases} 3x_1 & -2x_2 & = & 6 \\ 2x_1 & +x_2 & = & 11 \end{cases}$$

for x_1 and x_2.

Re-write the equations in matrix form $AX = B$:

$$\begin{pmatrix} 3 & -2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \end{pmatrix}$$

Illustration 1

Let's solve :

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for x_1 and x_2.

The determinant of A is :

$$\det A = 3 - (-2 \times 2) = 7$$

Illustration 1

Let's solve :

$$\begin{cases} 3x_1 & -2x_2 & = & 6 \\ 2x_1 & +x_2 & = & 11 \end{cases}$$

for x_1 and x_2.

A_1 is :

$$A_1 = \begin{pmatrix} 6 & -2 \\ 11 & 1 \end{pmatrix}$$

and $\det A_1 = 6 \times 1 - 11 \times (-2) = 28.$
So, as before :

$$x_1 = \frac{\det A_1}{\det A} = \frac{28}{7} = 4$$

Illustration 1

Let's solve :

$$\begin{cases} 3x_1 & -2x_2 & = & 6 \\ 2x_1 & +x_2 & = & 11 \end{cases}$$

for x_1 and x_2.

A_2 is :

$$A_2 = \begin{pmatrix} 3 & 6 \\ 2 & 11 \end{pmatrix}$$

and $\det A_2 = 3 \times 11 - 2 \times 6 = 21$.
So, as before :

$$x_2 = \frac{\det A_2}{\det A} = \frac{21}{7} = 3$$

Illustration 2

Let's solve :

$$\begin{cases} 2x & +4y & = & 5 \\ 2x & +y & = & 2 \end{cases}$$

for x and y.
Re-write the equations in matrix form $AX = B$:

$$\begin{pmatrix} 2 & 4 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

Illustration 2

Let's solve :

$$\begin{cases} 2x & +4y & = & 5 \\ 2x & +y & = & 2 \end{cases}$$

for x and y.
The determinant of A is :

$$\det A = 2 \times 1 - 2 \times 4 = -6$$

Illustration 2

Let's solve :

$$\begin{cases} 2x & +4y & = & 5 \\ 2x & +y & = & 2 \end{cases}$$

for x and y.
A_1 is :

$$A_1 = \begin{pmatrix} 5 & 4 \\ 2 & 1 \end{pmatrix}$$

and $\det A_1 = 5 \times 1 - 2 \times 4 = -3$.
So, as before :

$$x_1 = \frac{\det A_1}{\det A} = \frac{-3}{6} = 0.5$$

Illustration 2

Let's solve :

$$\begin{cases} 2x & +4y & = & 5 \\ 2x & +y & = & 2 \end{cases}$$

for x and y.

Illustration 2

Let's solve :

$$\begin{cases} 2x & +4y & = & 5 \\ 2x & +y & = & 2 \end{cases}$$

for x and y.
A_2 is :

$$A_2 = \begin{pmatrix} 2 & 5 \\ 2 & 2 \end{pmatrix}$$

and $\det A_2 = 2 \times 2 - 2 \times 5 = -6$.
So, as before :

$$x_2 = \frac{\det A_2}{\det A} = \frac{-6}{-6} = 1$$

Matrix algebra

- Addition is just summing elements
- Multiplication : remember the formula
- 0 matrix is filled with zeros
- 1 matrix has ones on its diagonal
- Division is not possible ! Use the inverse (if it exists) instead !

The inverse

- Check the determinant
- If $\det \neq 0$, the inverse exists !
- The inverse of A can be use to solve the system $AX = B$
- An other solution is $x_i = \frac{\det A_i}{\det A}$, following the Cramers' rule