Stokes flows and chaos - studying the dynamics of red blood cells

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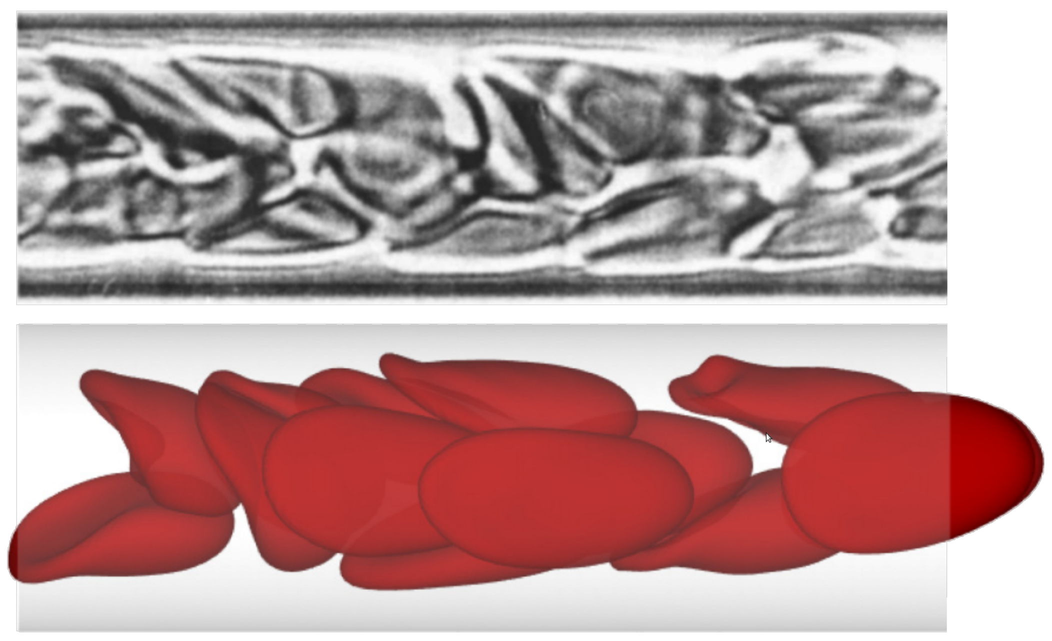
Contribution

The flow of red blood cells within cylindrical vessels is complex and irregular.
Understanding and modeling this flow is crucial in many applications, for instance oxygenation or drug delivery and assimilation.
The present contribution shows the main dynamical features of the blood flow, and provide an accurate reduced-order model of the dynamics of the blood cells in a stoke flow, [1].

Flow

Blood is a complex suspension whose primary components are red blood cells suspended in plasma. Red cells typically make up between 20% and 45% of whole blood by volume, depending upon the vessel size.



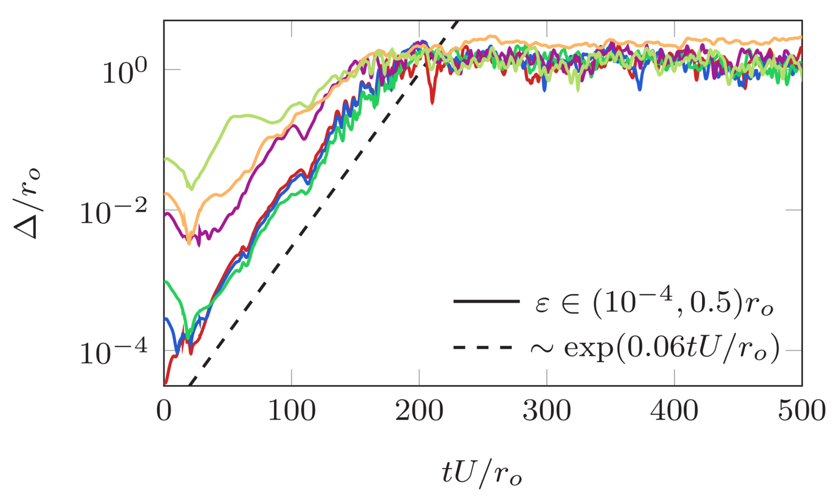
The configuration is a Stokes flow.
However, the dynamics of the cells are puzzling, as it exhibits features

- from a chaotic dynamics
- from a stochastic dynamics

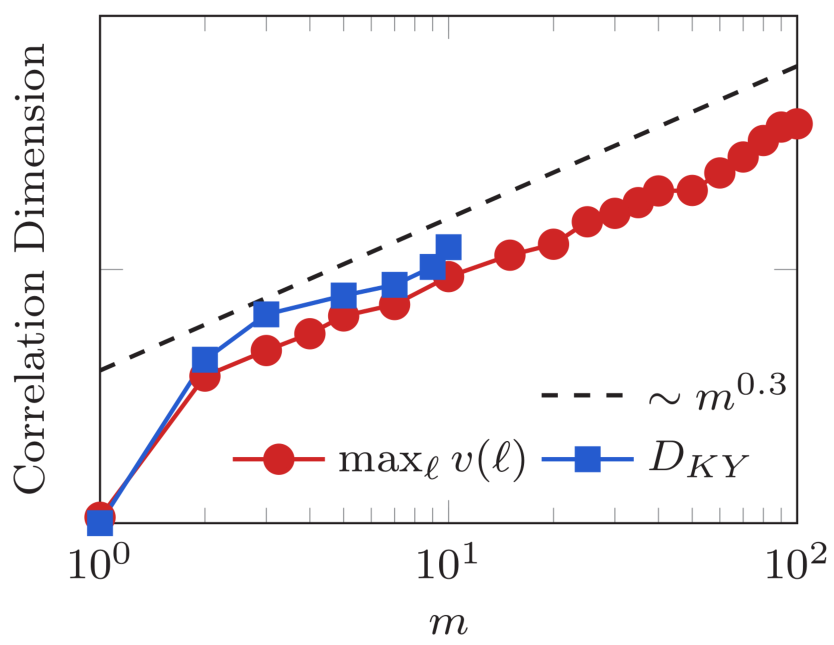
Phase-space reconstructions show that a low-dimensional attractor does not exist, so the observed long-time dynamics are effectively stochastic.

Stochastic or Hyperchaos?

The flow is sensitive to initial conditions.



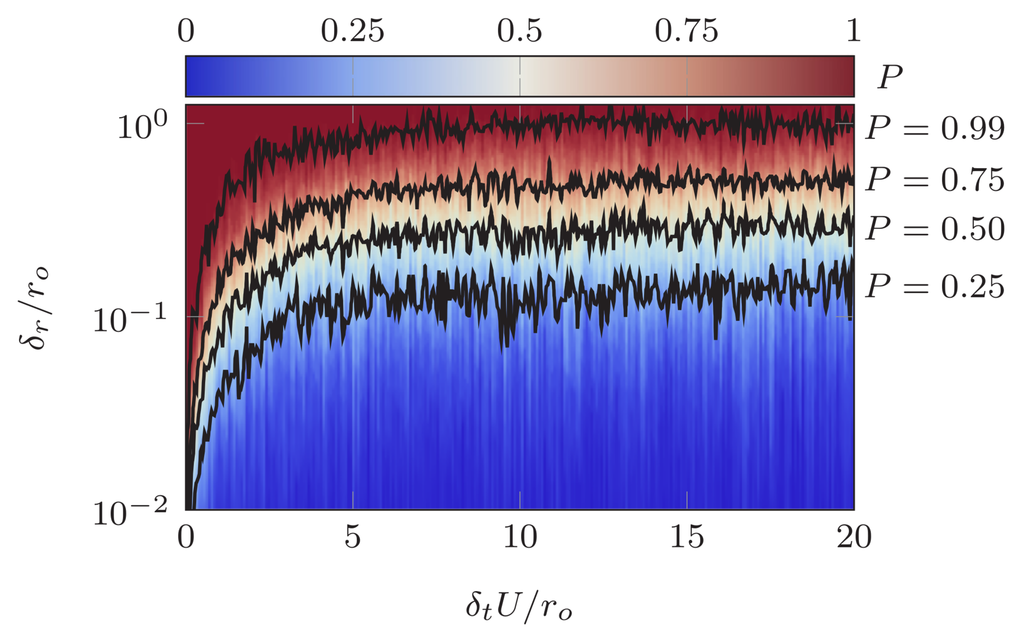
Sensitivity: Evolution of perturbations



Correlation dimensions & embedding dimensions

Due to the nature of the flow, chaos was anticipated.

The correlation dimension does not saturate, even for embedding dimensions larger than $m = 100$. A reduced order model based on a dynamical system cannot be properly constructed.



Space-time separations

Space-time separations, $\delta_r = |r_j - r_i|, \delta_t = |t_j - t_i|$ and the associated probabilities $P(\delta_r < X, \delta_t)$ saturate. This indicates:

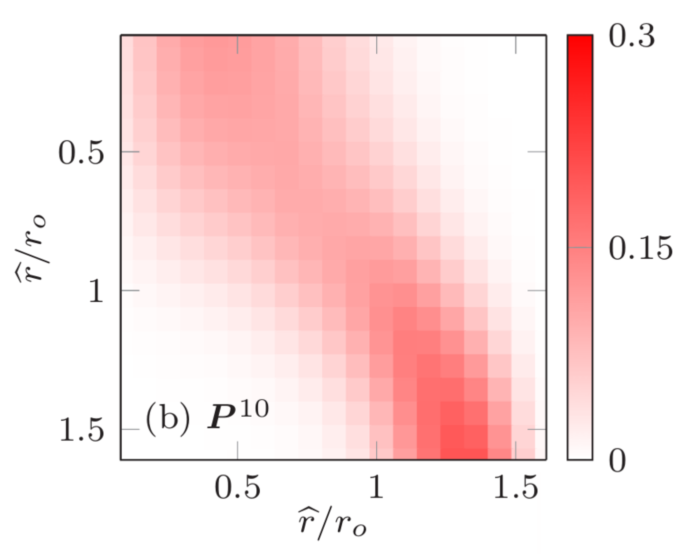
- absence of recurrences
- stochastic properties of the system, in particular the Markov property

Markov model

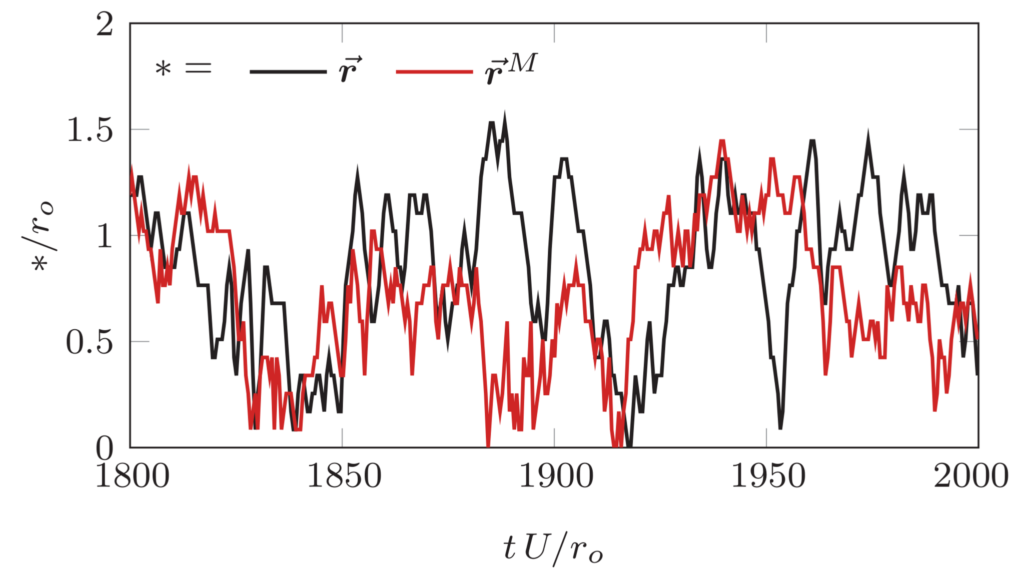
The high dimensions of the system and the absence of recurrent features lead to model the system based on its stochastic properties, [2].

A Markov model $\boldsymbol{P}$ is constructed, using the distance r of the cells *w.r.t.* the center of the vessel. $\boldsymbol{P}$ represents the probabilities of transition from a radius to another.

$$r^M_{\tau(i+1)} = \boldsymbol{P} r^M_{\tau i}$$



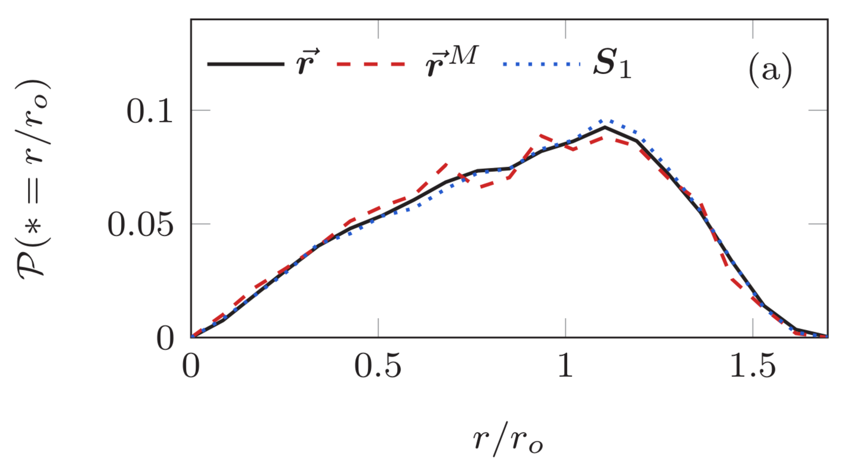
$\boldsymbol{P}^{10}$



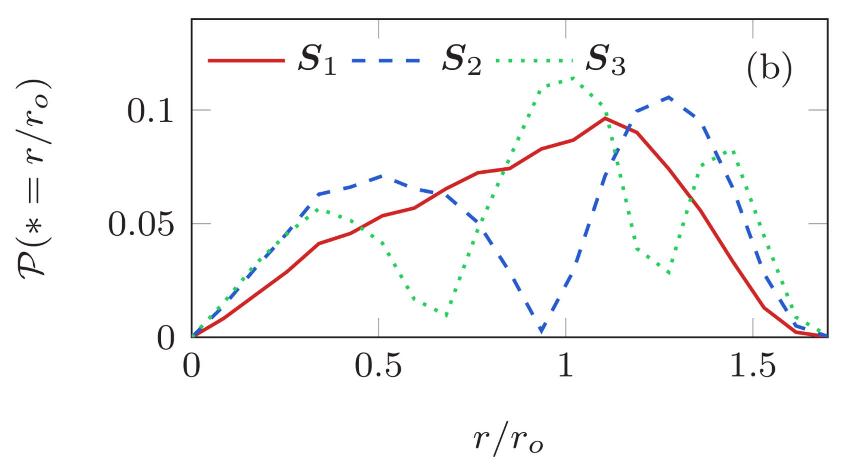
Full and Model flows

This allows to represent well the dynamics of the flow. The time series are well reconstructed and the first three moments of the distributions are preserved within 6% of accuracy.

Moment i	1 2	3	
Data	0.817	0.788	0.831
Model	0.795	0.753	0.782
error	2.76	4.52	5.97



distribution

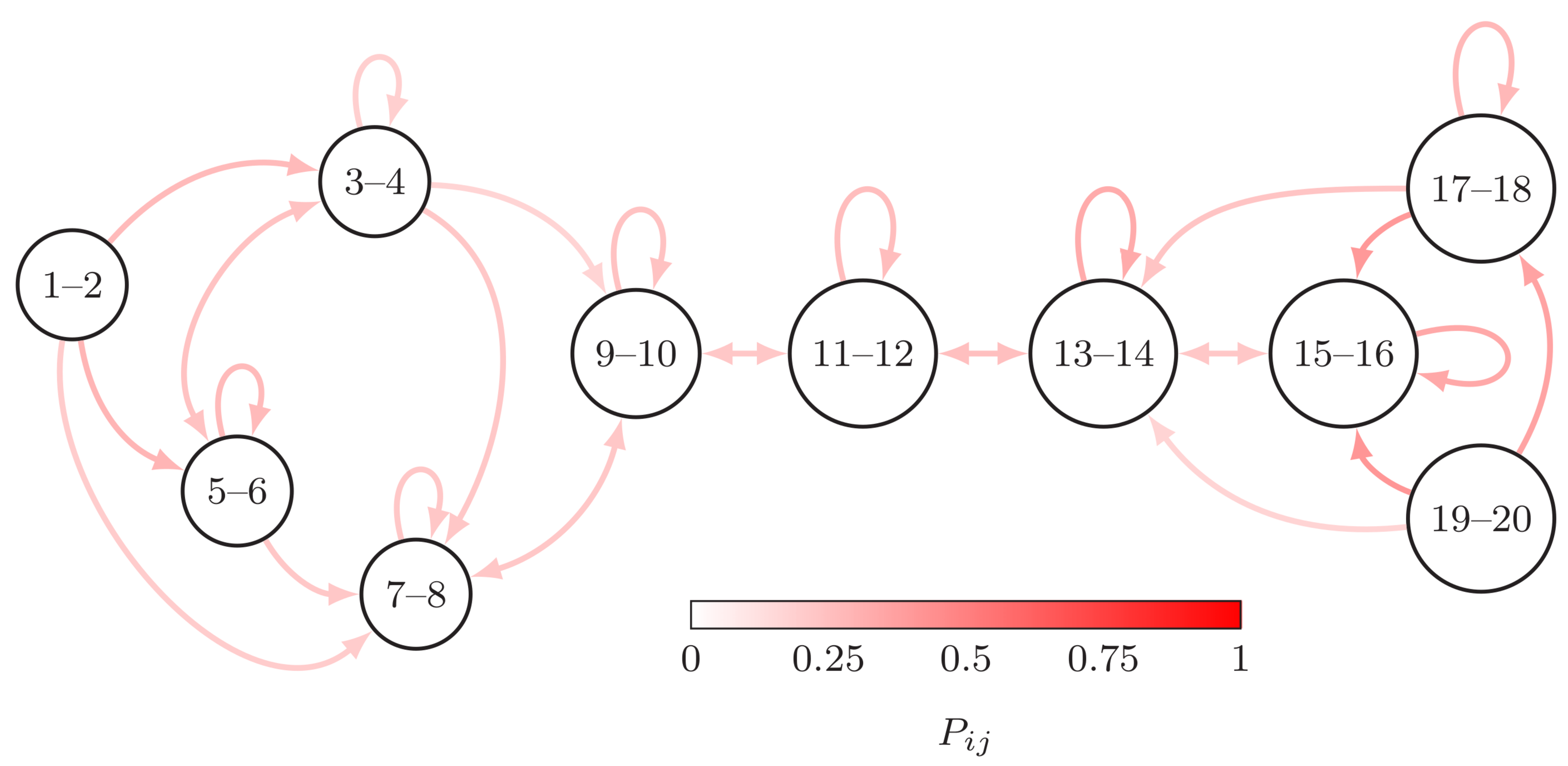


eigenmodes

It also exhibits features that were hard to distinguish in regular analysis.

There are recurrent patterns that were not discovered by Fourier or auto-correlation analysis, [1].
They are both identified by:

- the eigenmodes of the transition matrix $\boldsymbol{P}$
- transition probabilities between state clusters, [2].



Dynamics of transitions

In particular, there are two distinct modes of oscillations.

- close to the center
- close to the wall

References

[1] S. Bryngelson, F. Gueniat, and J. Freund., "Irregular dynamics of cellular blood flow in a model microvessel.," *Physical Review E*, vol. 1, no. 100, p. 012203, 2019.

[2] F. Guéniat, L. Mathelin, and M. Hussaini, "A statistical learning strategy for closed-loop control of fluid flows," *Theoritical Computational Fluid Dynamics*, vol. 30, pp. 497–510, December 2016.