SNAPSHOT-BASED FLOW ANALYSIS WITH ARBITRARY SAMPLING

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Summary In this paper, we present a method to extract Dynamic Mode Decomposition (DMD)-like modes from a dataset formed by snapshots taken at arbitrary times.

MOTIVATION

Several tools are readily available for analysis of unsteady fluid flows such as the celebrated *Proper Orthogonal Decomposition* (POD) or, the more recently introduced, Dynamic Modes Decomposition (DMD), [3]. Dynamic Modes Decomposition relies on a set of observable vectors acquired every Δt in time, *de facto* introducing a sampling frequency. The time step must be chosen small enough so as to resolve all time-scales of the underlying dynamics. The resulting ordered set of vectors defines a Krylov matrix from which the DMD algorithm extracts physically relevant modes.

However, this sampling process brings severe constrains on the measurement workflow. As an example, consider the typical situation where the observable is a two-dimensional 2-component velocity field acquired with a *Particle Imagery Velocimetry* (PIV) technique. Standard in PIV are fields of 1000×1000 pixels. Suppose the highest frequency of interest in the flow field is 200 Hz, a very mild assumption, the Shannon-Nyquist criterion imposes a sampling frequency above 400 Hz. With 12-bit images, the resulting data rate is then already above 1 Gb/s.

Further, if the Fourier spectrum is wide-banded, the timespan of the acquisition procedure has to be large. The combination of a high sampling frequency and a long acquisition sequence then quickly results in intractable constrains, both on measurement devices and computational resources.

Moreover, measurements of the observable may be corrupted, from external or intrinsic sources, say from the experimental setup or by failure in the data pre-processing. As a typical example, one can think of outliers in PIV. While it is still possible to account for corrupted data with some techniques, *e.g.* gappy-POD, dummy, physically-irrelevant, information might be introduced.

In this work, we use an approach to extract DMD-like modes which naturally copes with the limitations mentioned above.

ALGORITHM

The main objective is to derive an approximation of the flow field with a low-dimensional spectral decomposition. To this end, let consider the flow velocity vector $\boldsymbol{u}$ approximated under the form:

$$\boldsymbol{u}(\boldsymbol{x},t) \approx \sum_{k=1}^{N_m} \mathrm{e}^{i\,\sigma_k\,t}\,\boldsymbol{\phi}_k(\boldsymbol{x}) \equiv \sum_{k=1}^{N_m} \lambda_k^t \boldsymbol{\phi}_k(\boldsymbol{x}). \tag{1}$$

The methodology relies on a set of snapshots of observables $\{\boldsymbol{u}_{t_i}\}_{i=1}^N$, $t_i \in \mathbb{R}$, $\forall i$. These N snapshots are arranged in a Krylov-like matrix, $\mathbb{R}^{N_p \times N} \ni K := (\boldsymbol{u}_{t_1}\,\boldsymbol{u}_{t_2}\,\ldots\,\boldsymbol{u}_{t_N})$. Introducing the decomposition (1), K rewrites

$$K = M\,V + R \approx M\,V, \tag{2}$$

with R a residual matrix, $V \in \mathbb{C}^{N_m \times N}$ a Vandermonde-like matrix:

$$V := \begin{pmatrix} \lambda_1^{t_1} & \lambda_1^{t_2} & \ldots & \lambda_1^{t_N} \\ \lambda_2^{t_1} & \lambda_2^{t_2} & \ldots & \lambda_2^{t_N} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{N_m}^{t_1} & \lambda_{N_m}^{t_2} & \ldots & \lambda_{N_m}^{t_N} \end{pmatrix},$$

and $M \in \mathbb{C}^{N_p \times N_m}$ a matrix containing the spatial modes:

$$M := (\boldsymbol{\phi}_1\,\ldots\,\boldsymbol{\phi}_{N_m}).$$

To compute the decomposition (1), one then needs to determine matrices V and M. From the above, M can be approximated from:

$$M \approx K\,V^+,$$

with V^+ is the Moore-Penrose pseudo-inverse of V. Following similar lines as in [2], substitution of M in (2) yields $K \approx K\,V^+\,V + R$. Rearranging, it leads to

$$R \approx K\,\left(I_N - V^+\,V\right).$$

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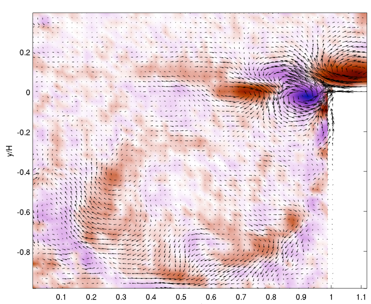
V is then finally determined as the matrix minimizing the Frobenius norm of R.

RESULTS

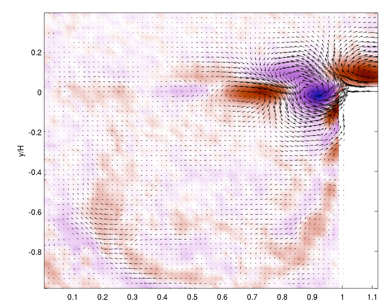
We now present results to illustrate the methodology. We focus on the turbulent flow over an open cavity. To investigate the flow dynamics, we rely on a set of 2D2C experimental, time-resolved, snapshots; see details on the setup and the flow configuration in [1]. The method presented above, hereafter referenced to as NU-DMD, was applied to snapshots taken at random within the full PIV dataset, hence with random time intervals between snapshots. PIV snapshots are coarsened from 10^6+ down to 7500 observables (vector components) and the full dataset contains 5500 snapshots.
The DMD algorithm is applied to the first N snapshots of the database while the NU-DMD algorithm relies on the same number N of snapshots taken at random. Comparison between DMD and NU-DMD is presented in Table 1.

N	12	28	38	65	90	200	500	1000
DMD	20.8	26.8	26.3	26.9	27.8	27.5	27.0	27.0
NU-DMD	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0

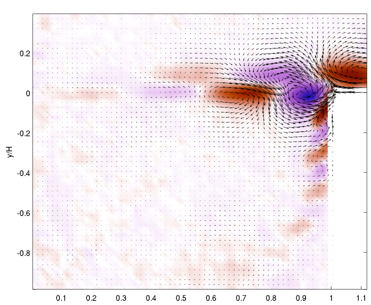
Table 1. Accuracy of the flow dominant mode frequency identification using different algorithms and sampling strategies. Notice that DMD fails to identify the dominant mode with less than about 90 snapshots. Identifying the precise frequency (27.0 Hz) requires even more snapshots while NU-DMD already succeeds with a few as $N = 12$.



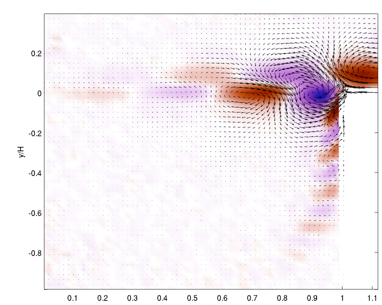
Dominant mode. NU-DMD, $N = 12$.



Dominant mode. NU-DMD, $N = 65$.



Dominant mode. NU-DMD, $N = 500$.



Dominant mode. DMD, $N = 500$.

It is seen that NU-DMD achieves rather similar results as the standard DMD approach. However, when the snapshots are not sampled uniformly in time, the DMD method can not be applied while the NU-DMD still identifies flow field dominant modes. Further, very few snapshots are necessary for the dominant features to emerge with this technique, as illustrated in the Table above. The NU-DMD method hence constitutes a valuable and widely applicable tool for analyzing physical systems from an observable dataset with very mild constrains (sampling strategy, missing snapshots, small dataset, etc.).

References

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