Lagrangian Coherent Structures in Open Cavity Flows

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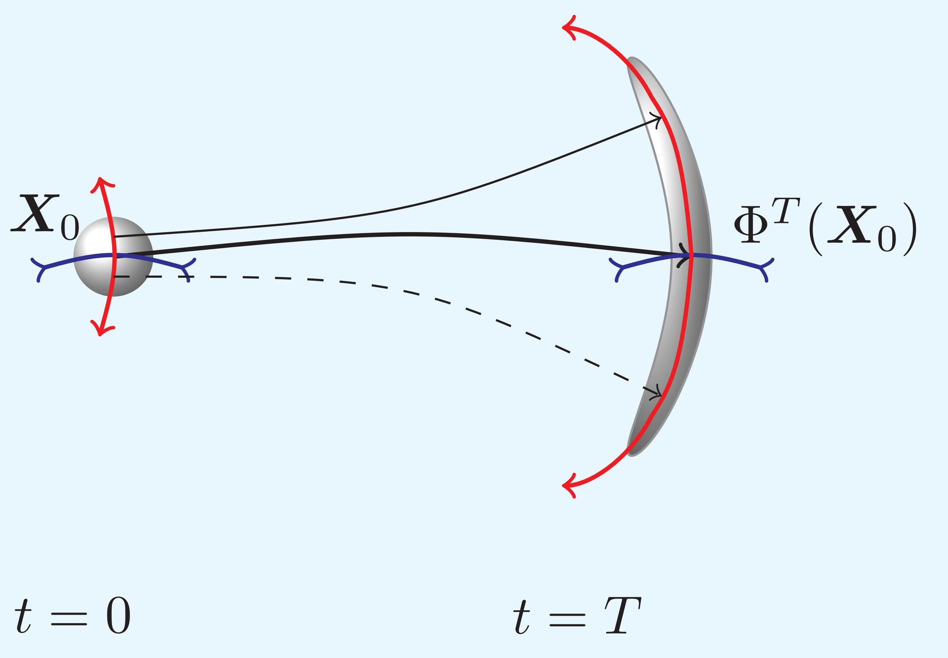
Lagrangian Coherent Structures

Let Φ^t be the flow which transports a particle from position $\boldsymbol{X}_0 \equiv \boldsymbol{X}(0)$ at time $t = 0$ to position $\boldsymbol{X}(t)$ at time t

$$\boldsymbol{X}(t) = \Phi^t(\boldsymbol{X}(0))$$



Stretching of fluid particles is evaluated by considering the distance $\delta\boldsymbol{X}(t)$ between two initially close particles:

$$\delta\boldsymbol{X}(T) = \frac{\mathrm{d}\Phi^T(\boldsymbol{X})}{\mathrm{d}\boldsymbol{X}}\,\delta\boldsymbol{X}(0) \equiv \boldsymbol{J}\,\delta\boldsymbol{X}(0)$$

where it appears that the fluid particle deformations can be quantified by the Cauchy-Green tensor [1, 2]:

$$\boldsymbol{C} = \boldsymbol{J}\cdot\boldsymbol{J}^\dagger, \qquad \boldsymbol{J}^\dagger \text{ transpose of } \boldsymbol{J}$$

The fluid particle deformation rate is driven by the largest eigenvalue, λ, of $\boldsymbol{C}$. λ is defined as the *Finite-Time Lyapunov Exponent* (FTLE).

Lagrangian Coherent Structures are defined as ridges of the FTLE field [1, 3]. In steady or time-periodic flows, LCS are invariant manifolds under Φ_t.

Algorithm

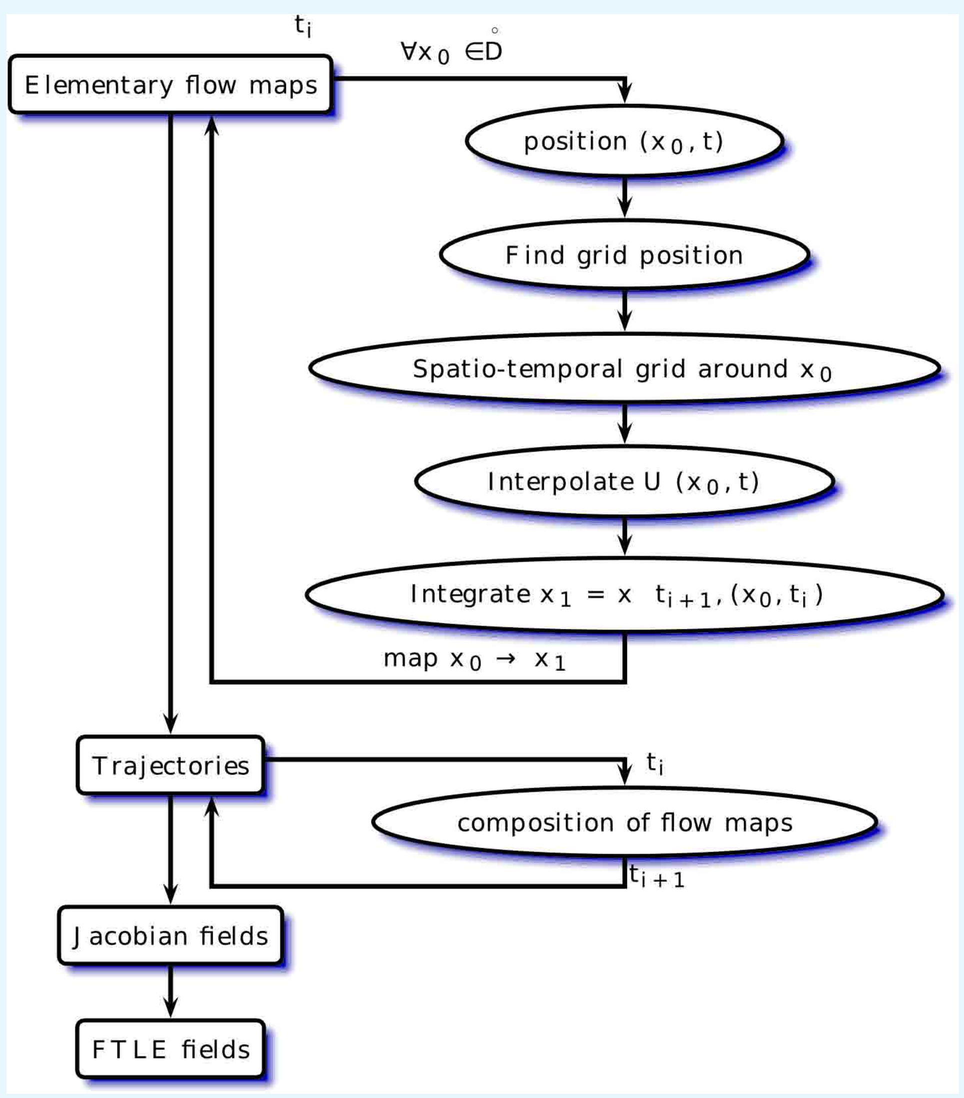
Trajectory in physical space seen as composition of elementary flows [4]

$$\Phi^T(\boldsymbol{X}(0)) = \boldsymbol{X}(T)$$

with

$$\Phi^T = \underbrace{\Phi^{dt} \circ \ldots \circ \Phi^{dt}}_{N} = \left(\Phi^{dt}\right)^N, \qquad T = N\,dt$$

NB : in an unsteady flow, Φ^{dt} changes with time.



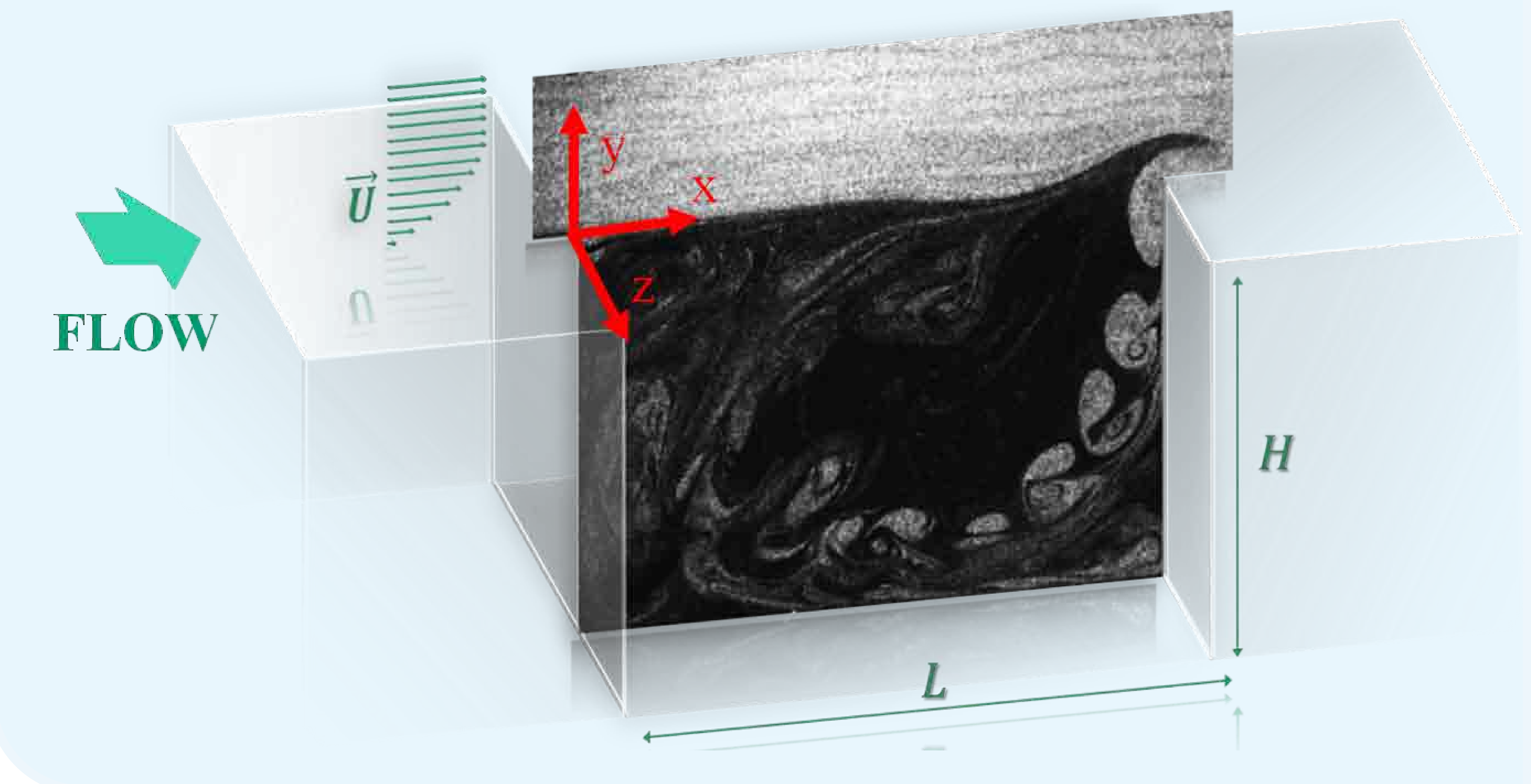
Bottlenecks of the algorithm:

- ✗ Elementary flow maps
- ✓ SIMD Vectorization – x100
- ✗ Composition of flow maps
- ✓ GPU Interpolation – x100

References

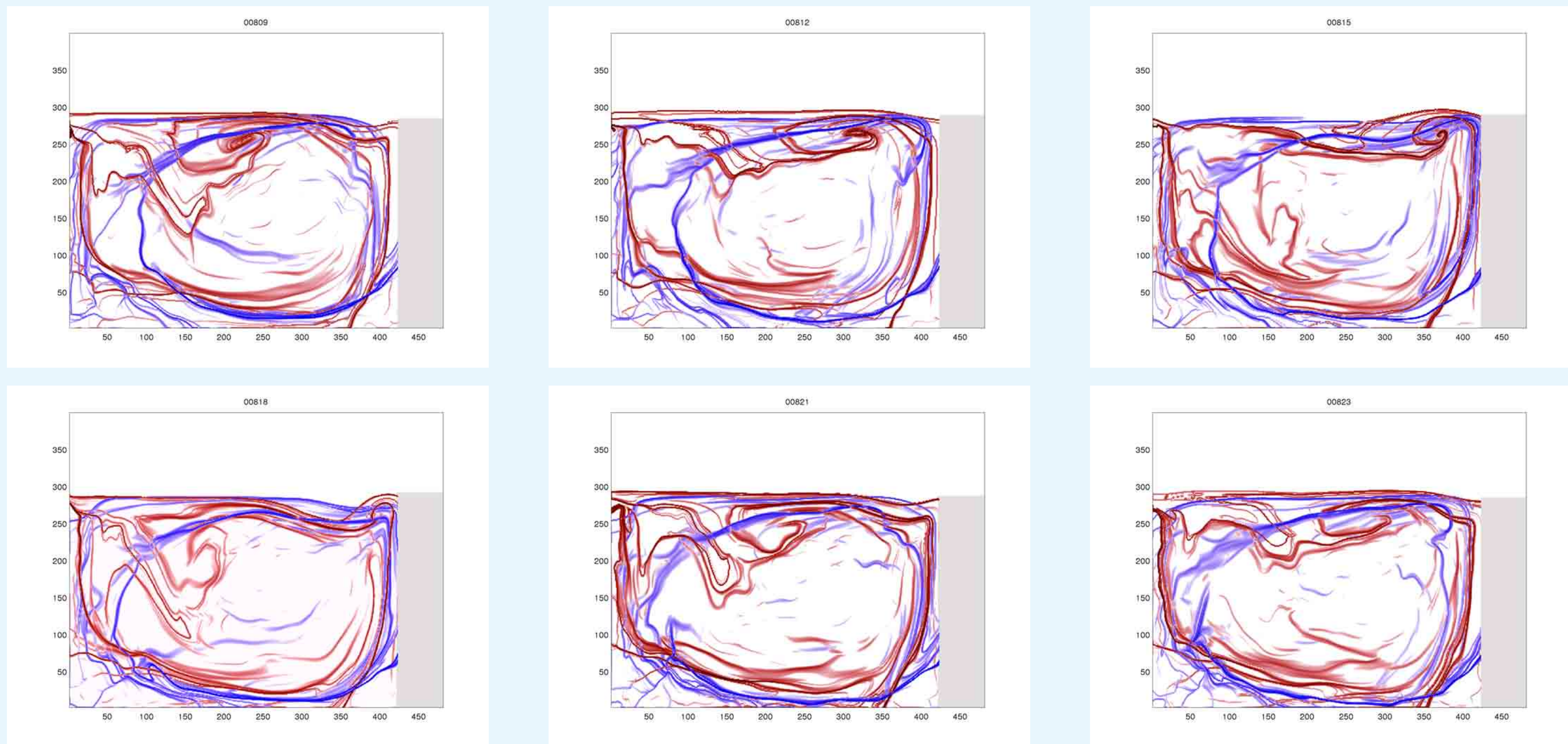
[1] Haller *et al.*, LCS and mixing in 2D turbulence **Chaos**, (2000),
[2] Haller, LCS from approximate velocity data, **PoF**, (2002)
[3] S.C. Shadden *et al.*, Definition and properties of LCS from FTLE in 2D aperiodic flows, **Physica D**, (2005)
[4] S.L. Brunton and C.W. Rowley, Fast computation of FTLE fields for unsteady flows, **Chaos** (2010)
[5] PJ. Schmid, Dynamic mode decomposition of numerical and experimental data, **JFM**, (2010).

Cavity flow



LCS time evolution

Horizon time $T = \ldots/f_a$, where f_a is the dominant frequency of the shear-layer oscillations.



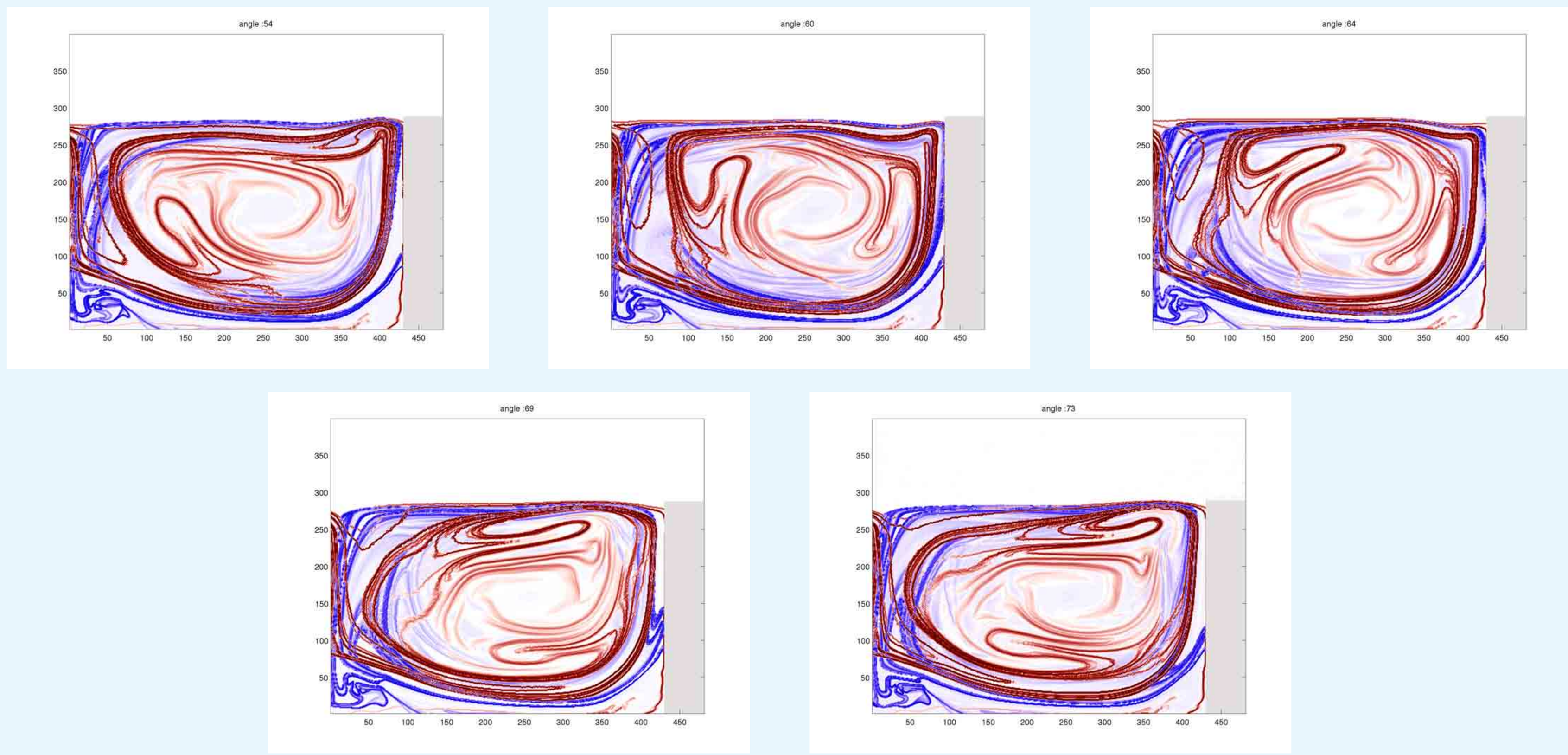
Fluid cannot cross LCS ⇒ material frontiers.
Lobe expansion/shrinkage violate flow incompressibility ⇒ hints of out-of-plane flow.

Dominant flow feature

Periodic dominant flow feature extracted by **Dynamic Mode Decomposition** [5]:

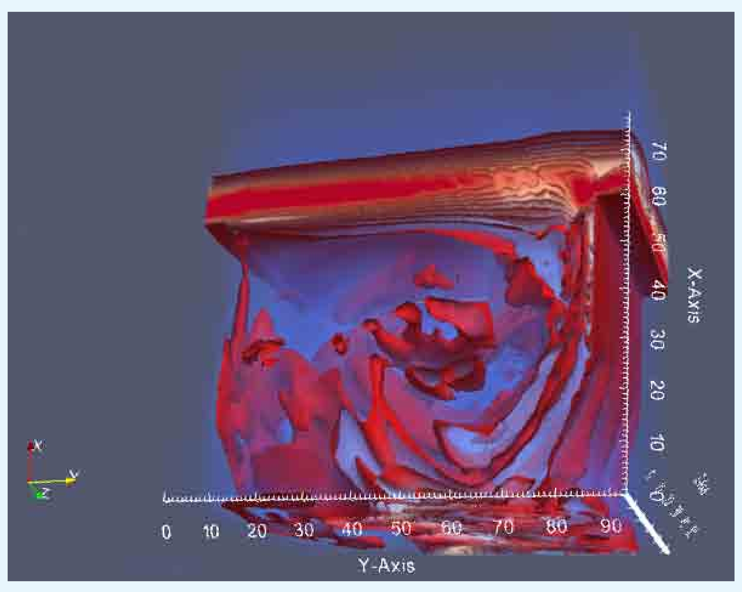
$$\boldsymbol{u}_{\boldsymbol{\omega}}(\boldsymbol{r}, t) = \bar{\boldsymbol{u}}(\boldsymbol{r}) + \mathcal{R}e\left(e^{i\omega t}\boldsymbol{\Phi}_\omega(\boldsymbol{r})\right)$$

where $\boldsymbol{\Phi}_\omega$ is the dominant shear-layer dynamic mode.



Horseshoe-like structures reveal potential mixing areas

From 2D to 3D LCS



3D direct numerical simulations of the flow reveal fully 3D LCS.

However, LCS computed on 2D PIV fields catch most of in-plane features.