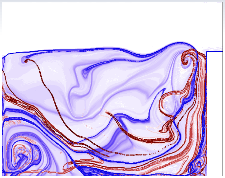

GPU and SIMD Acceleration for Identification of Lagrangian Coherent Structures.

Application to an Open Cavity Flow

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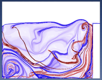

Hydrodynamic systems

Both open and confined flows are complex, and has potentially an infinite number of DoF.

... But coherent structures !

Von 'Heartman' street
Isla Socorro ($Re > 10^{10}$!).

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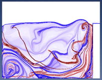

Lagrangian framework

In fluid mechanics :

$$\dot{\boldsymbol{X}} = \boldsymbol{u}\left(\boldsymbol{X}, t\right)$$

with $\boldsymbol{X} \in \mathcal{R}^3$
$\boldsymbol{u}$ comes from DNS or PIV measurement.

Or, $\nabla.\boldsymbol{u} = 0$ implies the system is conservative (within dynamical system frame).

If the system is autonomous or periodic, then the dynamic is driven by invariant manifolds.
When the system is non-autonomous, manifolds are no-more unique !

How find equivalent structures in unsteady flows ?

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Outline

1. Lagrangian Coherent Structures
   - → Presentation of LCS
   - → The Finite Time Lyapunov Exponent field
   - → Elementary flow map computation
   - → Computational improvements
2. Mixing features from a 2D experimental flows
   - → Qualitative results
   - → Reduce Order Model
3. Relevance of 2D analysis in 3D flows

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Lagrangian Coherent Structures and Finite-Time Lyapunov Exponent

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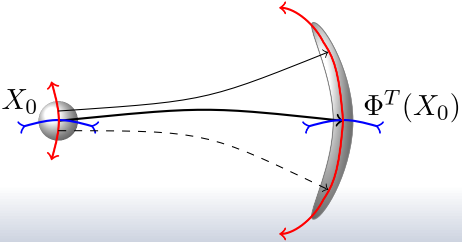
Material frontier

Dynamical flow : $\vec{\boldsymbol{X}}(t) = \Phi^t\left(\vec{\boldsymbol{X}}(0)\right)$

Invariant manifolds are invariant through the flow.

Consequently : such a manifold is a material frontier.



time : $t = 0$ time : $t = T$

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Stretching of fluid particles

How to identify these manifolds ?

Fluid particles are deformed by manifolds
$\rightarrow$ looking at stretching of fluid particles.

$$\delta X\left(T\right)=\frac{\mathrm{d}\Phi^{T}\left(X\right)}{\mathrm{d}X}\delta X\left(t_0\right)=J\delta X\left(t_0\right)$$

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Cauchy Green Tensor

For quantifying the stretching : the Cauchy Green Tensor :

$$\boldsymbol{C} = J \times J^{\dagger}$$

Then, the particule deformation rate is driven by the maximum eigenvalue of C, **i.e.** Finite-Time Lyapunov Exponent of the flow.

Haller,*et al.*, **Chaos**, (2000), **PoF**, (2002)

S.C. Shadden,*et al.*, **Physica D**, (2005)

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Lagrangian Coherent Structures

In autonomous/periodic system : ridges of the FTLE field are invariant manifolds.

In non autonomous system, there is non uniqueness of manifolds. Nevertheless, the ridges are still (most of the time) material frontier, and drive the mixing.

Ridges are called **Lagrangian Coherent Structures**.

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High Performance Computing

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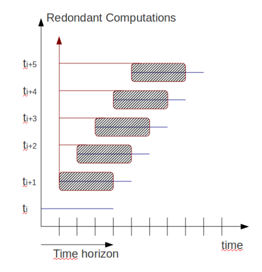
Intuitive (costly) way

Why we don't want computations of trajectories

For each point :
Jacobian of the flow
⇒ Cauchy-Green strain Tensor
⇒ 4 trajectories' computations ad minima around the point.

In a nutshell :
At time t, FTLE field in **1** point ⇒ **4** integrations.

Redondancy



How to avoid redondant computations of trajectories ?

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A smart way to estimate the Strain Tensor

Trajectory in physical space $\equiv$ composition of elementary flows.

$$\Phi_{t_A}^{t_C}\left(\boldsymbol{X}\left(t_A\right)\right)=\boldsymbol{X}\left(t_C\right)$$

with

$$\Phi_{t_A}^{t_C}=\Phi_{t_B}^{t_C}\circ\Phi_{t_A}^{t_B}$$

At time t, FTLE field in **1** point $\Rightarrow$ **1** "integration".

S.L. Brunton and C.W. Rowley, **Chaos** (2010)

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SIMD and GPU treatment

BottleNecks

1. Still Numerous particles.
2. SIMD implies Cartesian Grid, i.e. space increment has to be constant.
3. Elementary flow interpolation is time consuming.

Implemented solutions

1. SIMD – vectorization (x100).
2. Conformal transformation of the dataset.
3. Interpolation on GPU. (x100)

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Application to a cavity flow

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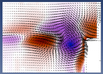
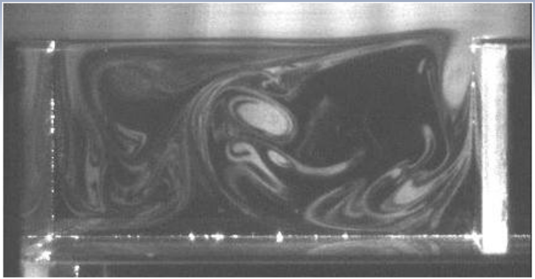


Introduction to Cavity flow



- $\mathrm{Re}_L = UL/\nu_{\mathrm{air}} > 10\,000$
- Dominant frequency in the flow : $St_L = fL/U \propto 1$ (≈ 20Hz)
- Sampling frequency : 250Hz

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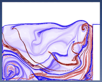
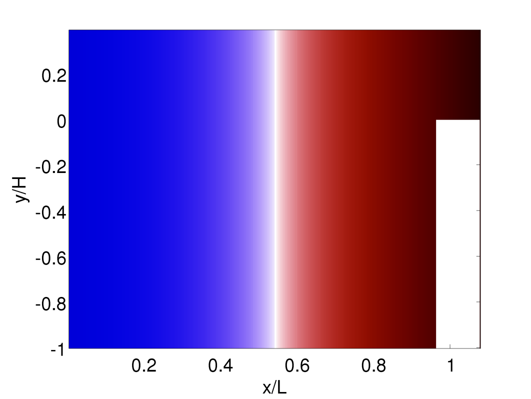


Dynamical Flow map : t_0



Representation of the flow over the x direction

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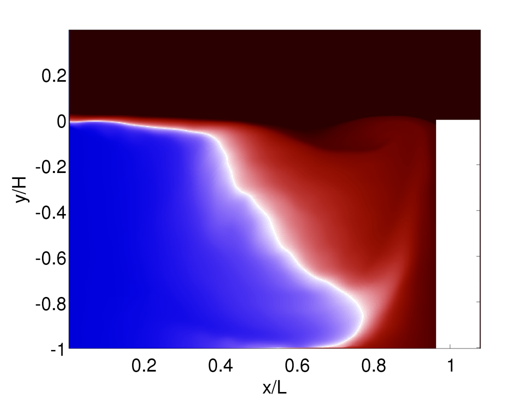
Dynamical Flow map : t_1



Representation of the flow over the x direction

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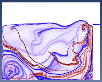
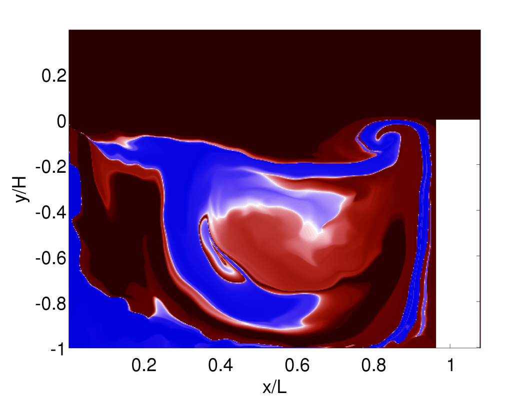


Dynamical Flow map : t_2



Representation of the flow over the x direction

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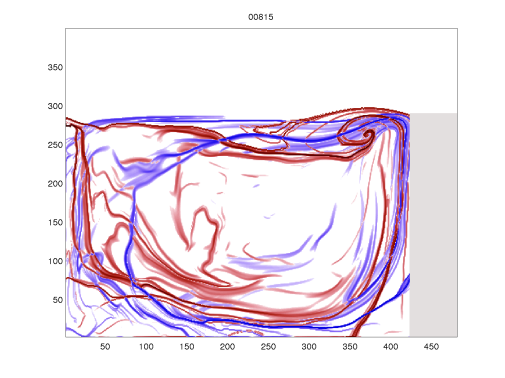
Forward and backward LCS

Qualitative results



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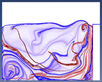
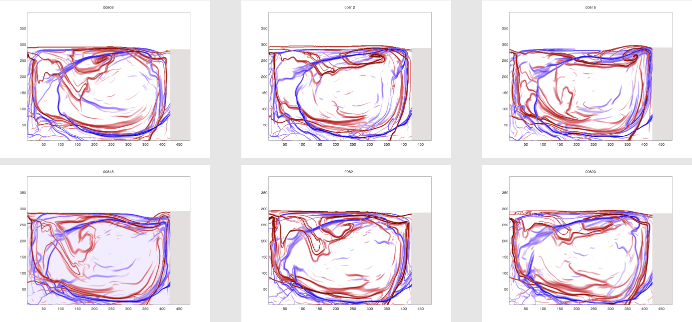


Forward and backward LCS

Qualitative results



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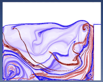

Simplification of the flow

DMD and Restricted Reduced Order Model

We build a synthetic flow, based on a Dynamical Modes Decomposition analysis of the dataset :

$$\boldsymbol{u}_{\boldsymbol{ROM}}\left(\boldsymbol{r},t\right)=\bar{\boldsymbol{u}}\left(\boldsymbol{r}\right)+\mathcal{R}e\left(\mathrm{e}^{i\omega t}\boldsymbol{\Phi}_{\omega}\left(\boldsymbol{r}\right)\right)$$

where $\boldsymbol{\Phi}_{\omega}$ is the dominant shear layer DMD mode.

The periodicity of the model is important : Now one can see LCS structures as invariant manifolds in a Poincarré' section.

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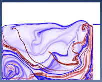
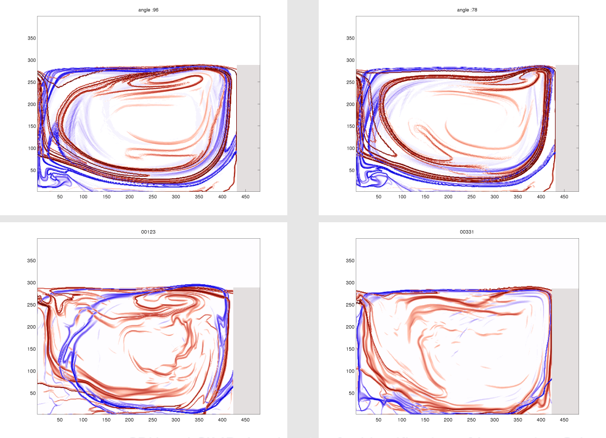
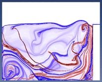


Verification of relevance

Comparison with real flows

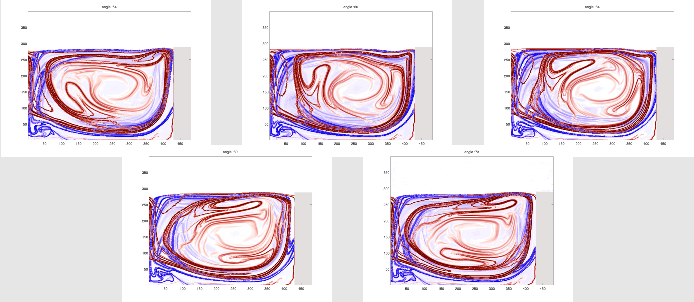
ROM flow structures (top) and real flow structures (bottom), with similar horizon.





Simplification of the flow (2/3)

Invariant manifolds of the model



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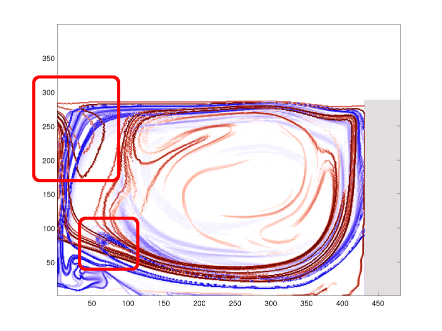
Simplification of the flow (3/3)

Horse-Shoes



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3D DNS comparison

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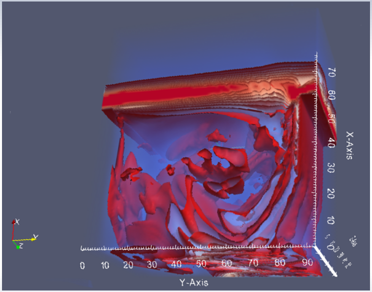
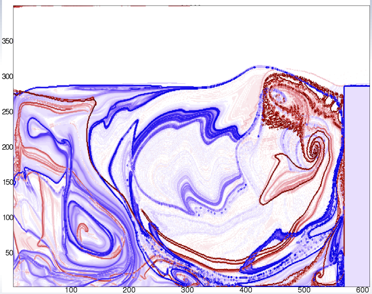
2D/3D FTLE fields



DNS



PIV

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Conclusions and openings

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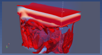

Conclusions and openings

- A very efficient implementation of the computations of Lagrangian Coherent Structures
- Efficent way to reveal mixing areas
- LCS features are captured by the Reduce Order Model
- Relevance of 2D fields for 3D cavity flows

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This is the end

Thank you for your patience and attentiveness !

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Annexes - Conformal transformations

Problem of the code : good only for regular mesh (for the SIMD)
Always true in experiences
Never true for DNS computations !
Solution : conformal transformations ≡ ad hoc **inversible** deformation of the data.

$$\Delta x_{i\ reg}/\vec{U}_{conf}\left(\boldsymbol{X}\right)=\Delta t=\Delta x_i/\vec{U}\left(\boldsymbol{X}\right)$$

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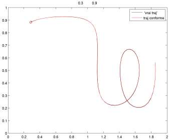
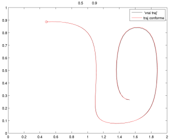
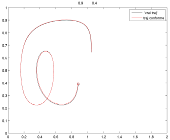
Annexes - Conformal transformations







Some examples in a chaotic but synthetic flow.

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