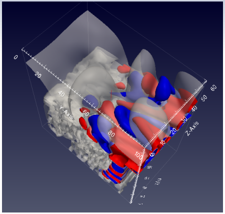

Investigating 3D features of an intermittent cavity flow

experimental and numerical analysis

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Actual dimension reduction

Coherent structures

Even if both open and confined flows are complex, and has potentially an infinite number of DoF, coherent structures (vortex, soliton, saturated instabilities) are present and it looks like it drives the dynamics.

It seems legit to look after a 'low' number of modes.

Von 'Heartman' street
Isla Socorro
($Re > 10^{10}$!).

Modal decomposition(s)

Suppose space and time separation and assume the existence of a basis, $\{\boldsymbol{\psi}_k(\boldsymbol{r})\}$ or $\{\alpha_k(t)\}$, to describe any flow realization

$$\boldsymbol{v}(\boldsymbol{r}, t) = \sum_{k \geq 1} \alpha_k(t) \boldsymbol{\psi}_k(\boldsymbol{r}) \simeq \sum_{k=1}^{N_m} \alpha_k(t) \boldsymbol{\psi}_k(\boldsymbol{r})$$

→ Proper Orthogonal Decomposition $(\alpha_k(t), \psi_k(\boldsymbol{r}))$
→ Global modes $(\omega, \psi_\omega(\boldsymbol{r}))$
→ Dynamical modes $(\omega, \psi_\omega(\boldsymbol{r}))$

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Outline

1. Modal decomposition
   - Dynamic Mode Decomposition (DMD)
   - Proper Orthogonal Decomposition (POD)
   - Spatial properties inheritance
2. Investigation of flow features
   - Comparison 2D POD/2D DMD
   - Divergence analysis
   - Comparison 2D DMD modes vs 3D DMD modes
3. Conclusion

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Dynamic Mode Decomposition

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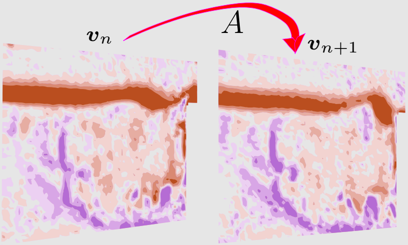
What are dynamic modes ?

Schmid *et al* (2008) *66th APS meeting* ; Rowley *et al*, (2009) *J. Fluid Mech.* ; Schmid, (2010) *J. Fluid Mech.*

→ Assume there exists an operator of evolution, A, such as the $\boldsymbol{v}_k$ are realisations of a *nonlinear* process.



→ Find a similar matrix to A. **Dynamic modes are defined as eigenvectors of A**, computed thanks to the similar matrix.

How to compute DMD modes ?

With $\boldsymbol{v}_{N+1} = \boldsymbol{v}_1 c_1 + \ldots + \boldsymbol{v}_N c_N$, then :

$$AK_1^N = K_1^N C = K_2^{N+1} \qquad \Rightarrow \qquad C = \begin{pmatrix} 0 & 0 & \ldots & 0 & c_1 \\ 1 & 0 & \ldots & 0 & c_2 \\ 0 & 1 & \ldots & 0 & c_3 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \ldots & 1 & c_N \end{pmatrix}$$

Eigenvalues of C are eigenvalues of U. Let ν be an eigenvector associated with the eigenvalue λ :

$$\begin{aligned} UK_1^N \nu &= K_1^N C \nu \\ &= K_1^N \lambda \nu \\ U \left(K_1^N \nu\right) &= \lambda \left(K_1^N \nu\right) \end{aligned}$$

Eigenvectors ψ of U are derived from eigenvectors ν of U : $\boldsymbol{\psi} \equiv K_1^N \boldsymbol{\nu}$

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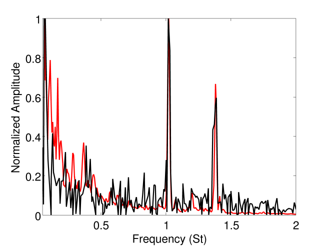
DMD spectrum



A power spectrum can be constructed on $\lambda = \|\lambda\| e^{\sqrt{-1}2\pi \mathrm{d}t}$ and $\|\psi\|$

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Proper Orthogonal Decomposition

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Proper Orthogonal Decomposition

A well-known method

We look for an orthonormal basis of spatial modes $\{\psi_i\}$, called **topos**, and temporal modes $\{\alpha_i\}$, called **chronos**, such as the average least-squares truncation error,

$$r_m = \sum_{k=0}^{t_N} \left\| \boldsymbol{u}(\boldsymbol{r}, t_k) - \sum_{i=0}^{m} \alpha_i(t_k) \boldsymbol{\psi}_i(\boldsymbol{r}) \right\|,$$

Chronos and topos are obtained through an Singular Values Decomposition of the dataset.

G. Berkooz, P. Holmes, JL. Lumley, (1993), *Annu. Rev. Fluid Mech.*
M. Bergmann, L. Cordier, JP. Brancher,(2007), *NNFM*

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Spatial properties inheritance

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Spatial properties of the flow

$$\boldsymbol{\nabla} \cdot \boldsymbol{v}\left(\boldsymbol{r}, t\right) = 0$$

By injecting the modal decomposition :

$$\begin{aligned} \boldsymbol{\nabla} \cdot \boldsymbol{v}\left(\boldsymbol{r}, t\right) &= \boldsymbol{\nabla} \cdot \left(\sum_i \alpha_i\left(t\right) \boldsymbol{\Phi_i}\left(\boldsymbol{r}\right)\right) \\ &= \sum_i \alpha_i\left(t\right) \boldsymbol{\nabla} \cdot \boldsymbol{\Phi_i}\left(\boldsymbol{r}\right) \\ &= 0. \end{aligned}$$

Then, remembering $\alpha_j\left(t\right)$ form an orthonormal basis, we have :

$$\begin{aligned} \int_{-\infty}^{\infty} \alpha_j\left(t\right) \sum_i \times \alpha_i\left(t\right) \boldsymbol{\nabla} \cdot \Phi_i\left(\boldsymbol{r}\right) \mathrm{d}t &= \int_{-\infty}^{\infty} \sum_i \alpha_j\left(t\right) \times \alpha_i\left(t\right) \boldsymbol{\nabla} \cdot \Phi_i\left(\boldsymbol{r}\right) \mathrm{d}t \\ &= \sum_i \boldsymbol{\nabla} \cdot \boldsymbol{\Phi_i}\left(\boldsymbol{r}\right) \int_{-\infty}^{\infty} \alpha_j\left(t\right) \times \alpha_i\left(t\right) \mathrm{d}t \\ &= \boldsymbol{\nabla} \cdot \boldsymbol{\Phi_j}\left(\boldsymbol{r}\right) \\ &= 0. \end{aligned}$$

Henceforward, as for the observable $\boldsymbol{v}$, the divergence of each mode is zero.

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Cavity flow

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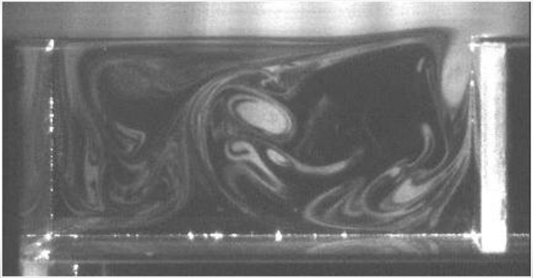
Cavity flow : Experimental setup



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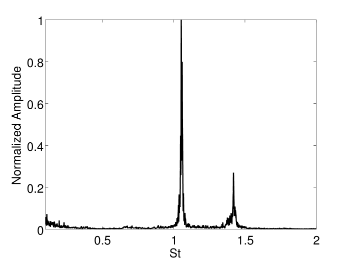
Cavity flow : Experimental setup

- Cavity length : $L = 100$ mm
- Geometric ratio : $L/H = 2$,
- Incoming velocity : $U = 1.90$ m/s
- Dataset : $N = 4096$ velocity fields.
- $\text{Re}_L = UL/\nu_{\text{air}} = 12\,700$
- Dominant frequencies in the flow $St_L = fL/U \propto 1$ (≈ 20Hz)
- Sampling frequency : 250Hz



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Proper Orthogonal Decomposition analysis

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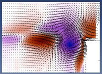
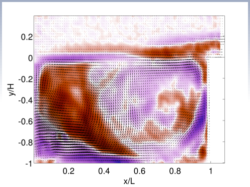
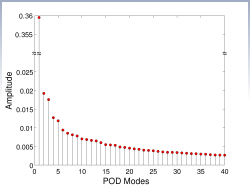


POD on the flow dataset

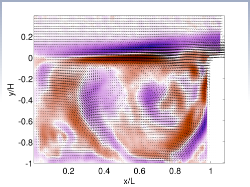
Topos n^o 4



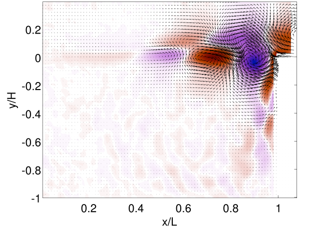
Modes Ranking



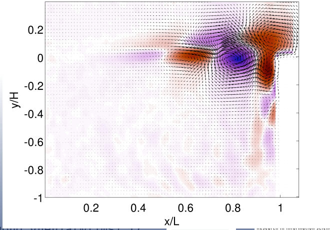
Topos n^o 5



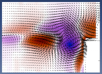
Topos n^o 2



Topos n^o 3

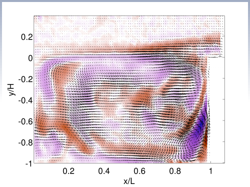


Dynamic Modes Decomposition analysis

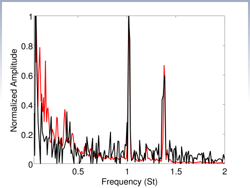


DMD on the flow dataset

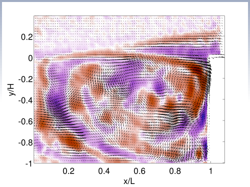
St= 0.08



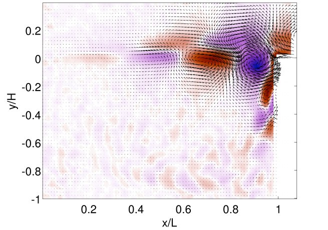
Spectra



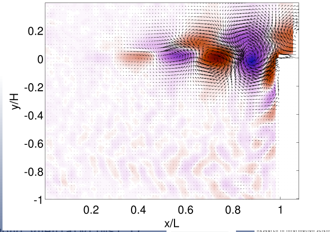
St= 0.13



St= 1.01



St= 1.38



POD-DMD comparison

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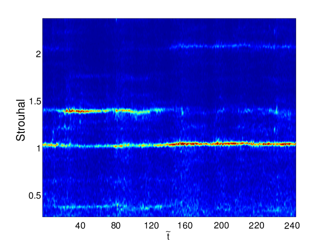
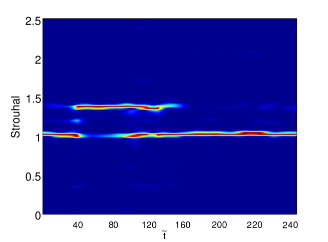
Detection of intermittency



sliding DMD



sliding Fourier analysis on a chronos

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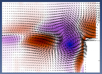
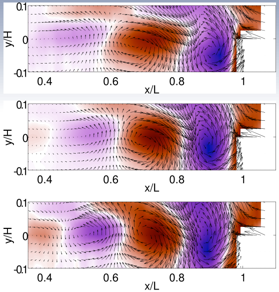


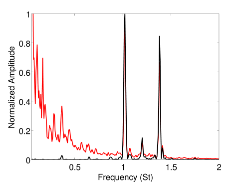
POD mixes lengths

DMD $St = 1$

POD n^o 2

DMD $St = 1.4$





$\lambda_{POD}/L = \frac{\gamma_1 \lambda_1 / L + \gamma_2 \lambda_2 / L}{\gamma_1 + \gamma_2}$,
where γ_1 and γ_2 are the respective amplitudes of DMD modes in the spectrum.

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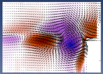
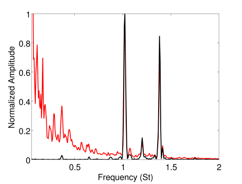


POD mixes lengths

TABLE: Frequency and wavelength comparison between POD analysis and DMD analysis

	St	λ/L
POD dominant mode	1.02 & 1.38	0.42
DMD mode $\mathbf{\Phi}_1$	1.02	0.49
DMD mode $\mathbf{\Phi}_2$	1.38	0.38

POD is not fitted for analysis of intermittency !



$\lambda_{POD}/L = \frac{\gamma_1 \lambda_1/L + \gamma_2 \lambda_2/L}{\gamma_1 + \gamma_2}$,
where γ_1 and γ_2 are the respective amplitudes of DMD modes in the spectrum.

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Divergence analysis

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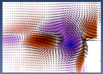
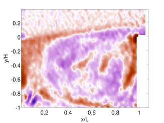
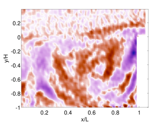


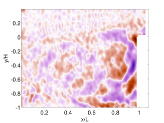
Divergence of main DMD modes



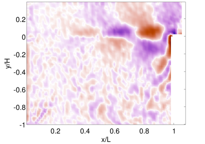
$St = 0$



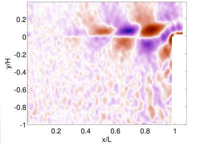
$St = 0.1$



$St = 0.3$



$St = 1.0$



$St = 1.4$

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3D DMD modes

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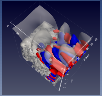
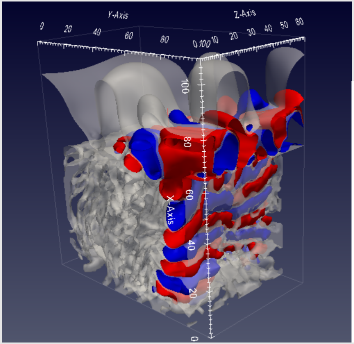
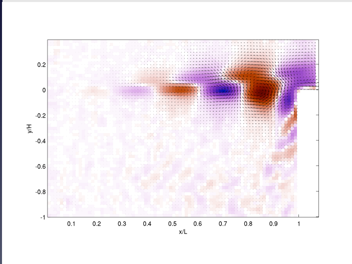


Comparison with shear layer DNS DMD mode





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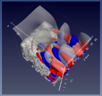
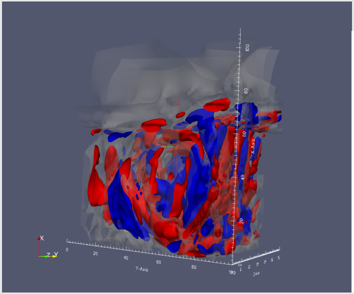
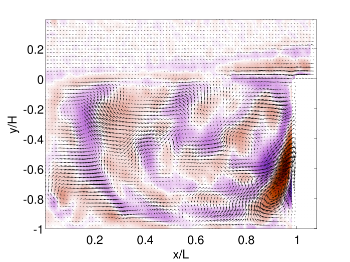


Comparison with inner flow DNS DMD mode





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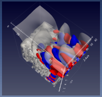

Conclusions

- Modes inherit spatial properties of the flow
- Application of DMD to an intermittent flow
- 3D features inferred from 2D experimental dataset
- Confirmation with a 3D DMD analysis on a numerical dataset

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This is the end

Thank you for your patience and attentiveness !

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